Please attempt ALL problems.

This is a “closed-book” examination.

You may NOT use your notes, nor any textbooks.

You must work independently. NO collaboration is allowed.

You may ask Dr. Curtright questions, however, especially if you think something is unclearly stated.

GOOD LUCK !!

“On my honor, I have neither received nor given aid on this exam.”

Name: ____________________________________________

Signature: _________________________________________

ID #: ____________________________________________
Exam total score (102=6x17 possible):

Please Note: In all of the following, bold letters denote vectors. Unit vectors along the $x$, $y$, and $z$ axes are denoted by $\mathbf{e}_x$, $\mathbf{e}_y$, and $\mathbf{e}_z$, respectively. A unit vector pointing radially away from the origin is denoted by $\mathbf{e}_r$. 
Multiple Choice Questions

Circle or underline the correct answer.

*Sometimes by accident or by design, none of the given answers may be correct. If you believe that to be the case, you should write-in what you believe to be the correct answer.*

[1] In the Figure above, the current in the long, straight wire is $I_1$ and the wire lies in the plane of the rectangular loop which carries current $I_2$. What is the magnitude and direction of the net force exerted on the loop by the magnetic field created by the wire?

1. $\mu_0 \ell I_1 I_2 \left( \frac{1}{2\pi (a+c)} - \frac{1}{2\pi c} \right) \hat{e}_{left}$
2. $\mu_0 \ell I_1 I_2 \left( \frac{1}{2\pi c} + \frac{1}{2\pi (a+c)} \right) \hat{e}_{left}$
3. $\mu_0 \ell I_1 I_2 \left( \frac{1}{2\pi c} - \frac{1}{2\pi (a+c)} \right) \hat{e}_{left}$
4. $\mu_0 \ell I_1 I_2 \left( \frac{1}{2\pi c} - \frac{1}{2\pi (a+c)} \right) \hat{e}_{right}$
5. $\mu_0 \ell I_1 I_2 \left( \frac{1}{2\pi c} + \frac{1}{2\pi (a+c)} \right) \hat{e}_{right}$

[2] An insulated conductor consists of a circular loop of radius $R$ and two straight long sections, as in the above Figure. The wire lies in the plane of the paper and carries a current $I$. What is the magnetic field at the center of the loop?

1. $\frac{\mu_0 I}{4\pi R} (2 - 2\pi) \hat{e}_{out \ of \ page}$
2. $\frac{\mu_0 I}{4\pi R} (2 - 2\pi) \hat{e}_{into \ page}$
3. $\frac{\mu_0 I}{4\pi R} (-2 + 2\pi) \hat{e}_{into \ page}$
4. $\frac{\mu_0 I}{4\pi R} (2 + 2\pi) \hat{e}_{out \ of \ page}$
5. $\frac{\mu_0 I}{2\pi R} \hat{e}_{into \ page}$
An infinitely long idealized coaxial cable consists of two very thin-walled conducting hollow concentric cylinders of radii $r_1$ and $r_2$. A short section of the coaxial cable of length $l$ is shown in the Figure below. Current $I$ goes in one direction down the length of the SURFACE of the inner cylinder and in the opposite direction on the SURFACE of the outer cylinder. The current density is distributed uniformly on the surface of either cylinder.

[3] Use Ampere’s Law to determine the strength of the magnetic field $B(r)$ for $r_1 < r < r_2$ where $r$ is the distance from the central axis of the cylinders. The result is:

\begin{align*}
(1) \quad B(r) &= \frac{\mu_0 I}{2\pi r} \\
(2) \quad B(r) &= \frac{\mu_0 I r}{2\pi r^2} \\
(3) \quad B(r) &= \frac{\mu_0 I r}{2\pi r_1^2} \\
(4) \quad B(r) &= \frac{\mu_0 I r}{\pi r_1^2 - \pi r_2^2} \\
(5) \quad B(r) &= \frac{\mu_0 I}{\pi r_2^2} \\
\end{align*}

[4] What is the magnetic field energy $dU_B$ in a concentric cylindrical shell volume element of length $l$, radius $r$, radial thickness $dr$, and volume $dV = 2\pi l r dr$?

\begin{align*}
(1) \quad dU_B &= \frac{1}{4\pi \mu_0} l \mu_0 I^2 dr \\
(2) \quad dU_B &= \frac{1}{4\pi \mu_0} l \mu_0 I^2 r^3 dr \\
(3) \quad dU_B &= \frac{1}{4\pi \mu_0} l \mu_0 I^2 r^3 dr \\
(4) \quad dU_B &= \frac{1}{\pi \mu_0} l \mu_0 I^2 r^5 dr \\
(5) \quad dU_B &= \frac{1}{\pi \mu_0} l \mu_0 I^2 dr \\
\end{align*}

[5] Integrate your answer to [4] to obtain the total magnetic field energy $U_B$ inside the coaxial cable for the section of length $l$.

(Hints: $\int \frac{dr}{r} = \ln r$, $\int r^3 dr = \frac{1}{4} r^4$, $\int \frac{dr}{r^2} = -\frac{1}{2r}$) The result is:

\begin{align*}
(1) \quad U_B &= \frac{1}{4\pi \mu_0} l \mu_0 I^2 \ln \left(\frac{r_2}{r_1}\right) \\
(2) \quad U_B &= \frac{1}{10\pi \mu_0} l \mu_0 I^2 \left(\frac{r_2^4}{r_1^4} - 1\right) \\
(3) \quad U_B &= \frac{1}{10\pi \mu_0} l \mu_0 I^2 \left(\frac{r_2^4}{r_1^4} - 1\right) \\
(4) \quad U_B &= \frac{1}{4\pi \mu_0} l \mu_0 I^2 \left(\frac{1}{r_1^4} - \frac{1}{r_2^4}\right) \\
(5) \quad U_B &= \frac{1}{2\pi^2} l \mu_0 I^2 \left(\frac{1}{r_1^4} - \frac{1}{r_2^4}\right) \\
\end{align*}

[6] Use your answer to [5] and $U_B = \frac{1}{2} LI^2$ to determine the self-inductance of the coaxial section of length $l$. The result is:

\begin{align*}
(1) \quad L &= \frac{1}{4\pi \mu_0} l \mu_0 \ln \left(\frac{r_2}{r_1}\right) \\
(2) \quad L &= \frac{1}{8\pi \mu_0} l \mu_0 \left(\frac{r_2^4}{r_1^4} - 1\right) \\
(3) \quad L &= \frac{1}{8\pi \mu_0} l \mu_0 \left(\frac{r_2^4}{r_1^4} - 1\right) \\
(4) \quad L &= \frac{1}{2\pi (r_2^4 - r_1^4)} l \mu_0 \left(\frac{r_2^4}{r_1^4} - 1\right) \\
(5) \quad L &= \frac{1}{2\pi} l \mu_0 \left(\frac{1}{r_1^4} - \frac{1}{r_2^4}\right) \\
\end{align*}
[7] Two single-turn circular loops of thin wire have radii $R$ and $r$, with $R \gg r$. The loops lie in the same plane and are concentric. Current $I$ flows around the smaller loop. What is the resulting magnetic flux $\Phi_B$ through the larger loop?

$$
\begin{align*}
(1) & \quad \mu_0 \pi r^2 / 2R \\
(2) & \quad \mu_0 \pi R^2 / 2r \\
(3) & \quad 2RI / \mu_0 \pi r^2 \\
(4) & \quad 2rI / \mu_0 \pi R^2 \\
(5) & \quad 0 
\end{align*}
$$

[8] A circular loop of area 1.0 $m^2$ is rotating about one of its diameters at $\omega = 100$ radians/sec with the axis of rotation perpendicular to a 0.10 $T$ magnetic field. If there are 1000 turns of insulated conducting wire around the loop, what is the maximum voltage induced in the wire?

$$
\begin{align*}
(1) & \quad 10 \text{ kV} \\
(2) & \quad 7.5 \text{ kV} \\
(3) & \quad 20 \text{ V} \\
(4) & \quad 7.5 \text{ V} \\
(5) & \quad 1.0 \text{ kV} 
\end{align*}
$$

[9] A 5.0 $mH$ inductor has a current going through it as given by $I = I_{\text{max}} \sin(\omega t)$ with $I_{\text{max}} = 10.00$ $A$ and $\omega = 100$ radians/sec. What is the voltage drop (in the direction of the current) in the inductor as a function of time?

$$
\begin{align*}
(1) & \quad 5.0V \cos(100t) \\
(2) & \quad 5.0V \sin(100t) \\
(3) & \quad 9.4V \cos(377t) \\
(4) & \quad 5.0V \cos(377t) \\
(5) & \quad 5.0V \sin(377t) 
\end{align*}
$$

[10] A series $RL$ circuit and a series $RC$ circuit have the same time constant. If the two circuits have the same resistance $R$, what is the value of $R$ in terms of $L$ and $C$?

$$
\begin{align*}
(1) & \quad R = \sqrt{LC} \\
(2) & \quad R = \sqrt{\frac{C}{L}} \\
(3) & \quad R = 1 / \sqrt{LC} \\
(4) & \quad R = \sqrt{LC} \\
(5) & \quad R = LC 
\end{align*}
$$

[11] A capacitor consisting of two very close parallel plates is initially uncharged. A current $I(t) = I_{\text{max}} \cos(\omega t)$ is switched on at time $t = 0$ and begins charging the lower plate of the capacitor. According to Gauss’s law, what is the upward electric field between the capacitor plates as a function of time? Let $A$ represent the area of each plate.

$$
\begin{align*}
(1) & \quad E = I_{\text{max}} \sin(\omega t) / \varepsilon_0 \omega A \\
(2) & \quad E = I_{\text{max}} \cos(\omega t) / \varepsilon_0 \omega A \\
(3) & \quad E = I_{\text{max}} \varepsilon_0 \omega A \sin(\omega t) \\
(4) & \quad E = I_{\text{max}} \varepsilon_0 \omega A \cos(\omega t) \\
(5) & \quad E = 0 
\end{align*}
$$

[12] The capacitor plates in the previous problem are separated by a distance $d$, with the same current flowing into it as before. What is the $rms$ voltage drop for this capacitor?

$$
\begin{align*}
(1) & \quad V_{\text{rms}} = dI_{\text{max}} / (\sqrt{2} \varepsilon_0 \omega A) \\
(2) & \quad V_{\text{rms}} = 0 \\
(3) & \quad V_{\text{rms}} = \sqrt{2}dI_{\text{max}} / (\varepsilon_0 \omega A) \\
(4) & \quad V_{\text{rms}} = AI_{\text{max}} / (\sqrt{2} \varepsilon_0 \omega d) \\
(5) & \quad V_{\text{rms}} = \sqrt{2}AI_{\text{max}} / (\varepsilon_0 \omega d) 
\end{align*}
$$
[13] The current in the circuit shown in the Figure above equals 0.5 of the maximum current at \( t = 8300 \, \mu s \). What is the smallest frequency of the generator which gives this current?

(1) \( f = 10.0 \, Hz \)  
(2) \( f = 630 \, Hz \)  
(3) \( f = 14.6 \, Hz \)  
(4) \( f = 91.7 \, Hz \)  
(5) \( f = 100 \, Hz \)

[14] An LCR circuit, shown in the above Figure, is subjected to a driving voltage \( V(t) = V_{\text{max}} \sin (\omega t) \). For fixed values of \( R, L, \) and \( C \), at what angular frequency, \( \omega \), will the current \( I(t) = I_{\text{max}} \sin (\omega t - \phi) \) have the largest value for \( I_{\text{max}} \)?

(1) \( \omega^2 = 1/\text{LC} \)  
(2) \( \omega^2 = \text{LC} \)  
(3) \( \omega^2 = L/C \)  
(4) \( \omega^2 = 1/\text{RC} \)  
(5) \( \omega^2 = R/L \)

For the next three problems, consider a toroid (a “donut”, or perhaps a “bagel”) of rectangular cross section as shown in the following Figures. The inner wall of the toroid is a distance \( a \) from the axis in the center of the toroid (i.e. from the center of the “hole in the donut”). The outer wall is a distance \( b \) from this central axis. The height of the toroid is \( H \). The toroid is tightly wound with a large number \( N \) of turns of an insulated wire carrying a current \( I \) in the direction shown. (Note: The front half of the donut is really there, but only shown as dashed lines.)
[15] Of the following Figures showing one closed magnetic field line (the thick-lined dark loops), which one represents the main features of the magnetic field in or around the toroid, especially the correct direction of $B$?
[16] If you use Ampere’s Law to determine $B(r)$ for $a < r < b$, the strength of the magnetic field INSIDE the toroid (the shaded portion) at a distance $r$ from the central axis, what is the result?

$\begin{align*}
1 \quad & B(r) = \frac{\mu_0 N I}{2 \pi r} \\
2 \quad & B(r) = \frac{\mu_0 N I}{\pi r^2} \\
3 \quad & B(r) = \frac{\mu_0 I}{\pi r} \\
4 \quad & B(r) = \frac{\mu_0 I}{2 \pi r} \\
5 \quad & B(r) = 2 \pi r \mu_0 NI
\end{align*}$

[17] Determine the total magnetic flux through the wire coiled around the toroid.
(Hints: $\int \frac{dr}{r} = \ln r$, $\int \frac{dr}{r} = -\frac{1}{r}$, $\int r dr = \frac{1}{2} r^2$.)

$\begin{align*}
1 \quad & \Phi = \frac{1}{2 \pi} \mu_0 N^2 IH \ln (b/a) \\
2 \quad & \Phi = \frac{1}{2 \pi} \mu_0 IH \ln (b/a) \\
3 \quad & \Phi = \frac{1}{2 \pi} \mu_0 N^2 IH \left(\frac{b-a}{ab}\right) \\
4 \quad & \Phi = \frac{1}{2 \pi} \mu_0 IH \left(\frac{b-a}{ab}\right) \\
5 \quad & \Phi = \pi \mu_0 IH \left(b^2 - a^2\right)
\end{align*}$