Please attempt ALL problems.

This is a “closed-book” examination.

You may NOT use your notes, nor any textbooks.

You must work independently. NO collaboration is allowed.

You may ask Dr. Curtright questions, however, especially if you think something is unclearly stated.

GOOD LUCK !!

“On my honor, I have neither received nor given aid on this exam.”

Name: ____________________________________________
Signature: _______________________________________
ID #: ________________________________________
Multiple choice total score (70 possible, 14 questions, 5 points each):

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Essay problem scores (30 possible, 1 question):

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Exam total score (100 possible):

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"Essay" Problem. Show all your work on this problem. Suppose one mole of an ideal monatomic gas undergoes reversible pressure and temperature changes, as shown in the following figure, such that \( PV = RT \) always holds. Calculate the change in entropy for the gas for the complete process \( A \to B \to C \).

In general we have

\[ \Delta S = \int dS = \int \frac{dQ}{T} = \int \frac{dU}{T} + \int \frac{dW}{T} \]

and for one mole of an ideal gas

\[ \Delta S_{\text{ideal gas}} = \int \frac{C_V dT}{T} + \int \frac{PdV}{T} = C_V \int \frac{dT}{T} + R \int \frac{dV}{V} \]

Now, we could just evaluate these integrals between limits corresponding to \( A \) and \( C \) in the figure. So

\[ \Delta S_{A\to C} = C_V \ln \left( \frac{T_C}{T_A} \right) + R \ln \left( \frac{V_C}{V_A} \right) \]

From the data shown in the figure, \( \frac{T_C}{T_A} = 2 = \frac{p_a}{p_a} \) and \( 2 \frac{V_A}{V_A} = \frac{R T_C}{R T_A} = \frac{T_C}{T_A} = 2 \), or \( \frac{V_C}{V_A} = 1 \). Thus

\[ \Delta S_{A\to C} = C_V \ln 2 \]

For one mole of a monatomic gas, \( C_V = \frac{5}{2} R \), so

\[ \Delta S_{A\to C} = \frac{3}{2} R \ln 2 \]

Alternatively, we may compute the entropy changes for the sub-processes shown as

\[ \Delta S_{A\to B} = \int_A^B \frac{dQ}{T} = \int_A^B \frac{C_p dT}{T} = \frac{5}{2} R \int_A^B \frac{dT}{T} = \frac{5}{2} R \ln \left( \frac{T_B}{T_A} \right) = \frac{5}{2} R \ln 2 \]

and

\[ \Delta S_{B\to C} = \int_B^C \frac{dW}{T} = \int_B^C \frac{dV}{V} = R \ln \left( \frac{V_C}{V_B} \right) = R \ln \frac{1}{2} = -R \ln 2 \]

where we used \( \frac{V_C}{V_B} = \frac{1}{2} \). So we again obtain

\[ \Delta S_{A\to C} = \Delta S_{A\to B} + \Delta S_{B\to C} = \frac{5}{2} R \ln 2 - R \ln 2 = \frac{3}{2} R \ln 2 \]
Multiple Choice Questions

Circle or underline the correct answer.

Sometimes by accident or by design, none of the given answers may be correct. If you believe that to be the case, you should write-in what you believe to be the correct answer.

[1] Moving along a streamline in a non-compressible, non-viscous (i.e. no friction!), flowing fluid, which of the following is guaranteed by the conservation of both mass and energy not to change from point to point? (In these expressions, $g$ is the height of the point in the streamline, $v$ is the speed of the fluid at that point, $P$ is the pressure, $A$ is the cross-sectional area of the local tube of streamlines, and $\rho$ is the (constant) density of the fluid. And of course, $g$ is the acceleration of gravity.)

(a) $P$
(b) $P + \rho g$
(c) $P + \rho g + \nu^2/2$
(d) $P + \nu^2/2$
(e) $\nu A^2$

[2] A cubical swimming pool is of width $w$, depth $w$, and length $w$. It is completely filled with water of density $\rho$. What is the magnitude of the total force exerted by the water against one of the sides of the pool? (Atmospheric pressure today is $P_0$, and gravitational acceleration is $g$)

(a) $F = P_0 w^2 + \frac{1}{2} \rho gw^3$
(b) $F = P_0 w^2 + \rho gw^3$
(c) $F = \rho gw^3$
(d) $F = \frac{1}{2} \rho gw^3$
(e) $F = P_0 w^2$

[3] A body of mass $m$ and specific heat $c$ initially $(t = 0)$ at a temperature $T_i$ cools by convection and radiation in a room where the temperature is $T_0$, with $T_i > T_0$. The body obeys Newton’s law of cooling, given by $dQ/dt = -hA(T - T_0)$, where $A$ is the surface area of the body and $h$ is a constant (the “surface coefficient of heat transfer”). What is the rate of heat exchanged by the body with its surroundings at time $t$?

(a) $dQ(t)/dt = (T_0 - T_i) \frac{mc}{h}$
(b) $dQ(t)/dt = -hA(T_i - T_0) \exp \left( \frac{hA t}{mc} \right)$
(c) $dQ(t)/dt = -hAT_0 + T_0 \frac{hA t}{mc}$
(d) $dQ(t)/dt = -hAT_i - T_0 \exp \left( -\frac{hA t}{mc} \right)$
(e) $dQ(t)/dt = -hAT_0 \exp \left( -\frac{hA t}{mc} \right)$

[4] Suppose one mole of a not-quite-ideal gas is described by the Clausius equation of state: $P(V - V_0) = RT$. Here $V_0$ is a fixed constant representing the net volume of all the molecules in the gas. What is the work done by the gas if it expands isothermally from volume $V_1$ to volume $V_2$?

(a) $\Delta W = RT \ln \left( \frac{V_0 - V_0}{V_1 - V_0} \right)$
(b) $\Delta W = RT \ln \left( \frac{V_1}{V_0} \right)$
(c) $\Delta W = RT \left( \frac{V_2}{V_0} - \frac{V_1}{V_0} \right)$
(d) $\Delta W = RT \left( \frac{V_2}{V_1} \right)$
(e) $\Delta W = 0$

[5] In one model of a solid, the material is assumed to consist of a regular array of atoms in which each atom has a fixed equilibrium position and is connected by springs to its neighbors. Each atom can vibrate in the $x$, $y$, and $z$ directions. The total energy of an atom in this model is $E = \frac{1}{2} m v_x^2 + \frac{1}{2} m v_y^2 + \frac{1}{2} m v_z^2 + \frac{1}{2} K x^2 + \frac{1}{2} K y^2 + \frac{1}{2} K z^2$. What is the average kinetic energy of an atom in the solid when the temperature is $k T$?

(a) $\frac{1}{2} kT$  (b) $3kT$  (c) $\frac{1}{2} kT$  (d) $2kT$  (e) $\frac{1}{2} kT$
[6] Two moles of an ideal monatomic gas, at an initial volume \( V_1 = 50 \text{ liters} \), follows the cycle shown in the following Figure. All processes are reversible, with the system always described by the ideal gas law. What is the net total amount of work \( \Delta W \) done by the gas on its surroundings during each cycle?
(a) \( 2.5 \times 10^3 \text{ J} \)
(b) \( 2.5 \times 10^4 \text{ J} \)
(c) 0.0 J
(d) \( 5.0 \times 10^3 \text{ J} \)
(e) \( 5.0 \times 10^4 \text{ J} \)

[7] Two moles of an ideal monatomic gas follows the cycle shown in the above Figure. What is the amount of heat flowing into the gas during each cycle? (Sum only the positive \( \Delta Q \)'s, not the negative ones!)
(a) \( 1.6 \times 10^4 \text{ J} \)
(b) \( 3.3 \times 10^3 \text{ J} \)
(c) \( 2.5 \times 10^4 \text{ J} \)
(d) \( 7.5 \times 10^3 \text{ J} \)
(e) \( 5.0 \times 10^4 \text{ J} \)

[8] At a depth \( h = 25 \text{ meters} \) below the surface of the sea (water density \( \rho = 1025 \text{ kg/m}^3 \)), where the temperature is \( T_C = 5.0^\circ C \), a diver exhales an air bubble having a volume of \( V_0 = 10. \text{ cm}^3 \). If the surface temperature of the sea is \( T_D = 20.\text{^\circ C} \), what is the volume of the air bubble right before it breaks the surface, where \( P = 1 \text{ atm} = 1.01 \times 10^5 \text{ N/m}^2 \)? (Neglect any absorption of the gas in the bubble by the surrounding water.)

(a) 37 cm³
(b) 3.5 cm³
(c) 6.6 cm³
(d) 14 cm³
(e) 16 cm³

Solution: \( PV = nRT \), so with \( nR \) constant, we have
\[
V_{\text{surface}} = \frac{T_{\text{surface}}}{P_{\text{depth}}} \cdot \frac{P_{\text{depth}}}{T_{\text{depth}}} V_{\text{depth}} = \frac{293}{1.01 \times 10^5} \cdot \frac{3.52 \times 10^5}{278} \times 10. \text{ cm}^3 = 36.7 \text{ cm}^3
\]

using \( P_{\text{depth}} = P_{\text{surface}} + \rho g z = 1.01 \times 10^5 + 1025 \times 9.8 \times 25 = 3.52 \times 10^5 \text{ N/m}^2 \).

[9] One mole of an ideal diatomic gas is enclosed in a cylinder that has a movable piston on top. The piston has a mass \( m \) and an area \( A \) and is free to slide up and down, keeping the pressure of the gas constant. How much heat is absorbed by the gas as the temperature is slowly raised from \( T_1 \) to \( T_2 \)?
(a) \( \Delta Q = \frac{3}{2} R (T_2 - T_1) \)
(b) \( \Delta Q = R T_1 (1 + \ln(T_2/T_1)) \)
(c) \( \Delta Q = \frac{3}{2} R (T_2 - T_1) \)
(d) \( \Delta Q = R T_2 (1 + \ln(T_1/T_2)) \)
(e) \( \Delta Q = \frac{3}{2} R (T_2 - T_1) \)
[10] In a time \( t \), \( N \) vertically falling hailstones strike a flat horizontal glass window skylight of area \( A \). Each hailstone has a mass \( m \) and a speed \( v \). If the collisions are elastic (i.e. no energy is lost), what is the average pressure on the window?
(a) \( P = \frac{N}{2} \sqrt{2}mv \)
(b) \( P = \frac{N}{4} mv \)
(c) \( P = \frac{N}{4} 4mv \)
(d) \( P = \frac{N}{1} 2mv \)
(e) \( P = 0 \)

[11] Assume the atmosphere is at constant temperature, \( T \), and that the acceleration of gravity is constant, \( g \). What fraction of molecules of individual mass \( m \) in the gas of the atmosphere lie between heights \( h \) and \( 2h \)?
(a) \( 1 - \exp\left(-\frac{mgh}{kT}\right) \)
(b) \( \left(1 - \exp\left(-\frac{mgh}{kT}\right)\right) \exp\left(-\frac{mgh}{kT}\right) \)
(c) \( 1 - \exp\left(-\frac{2mgh}{kT}\right) \)
(d) \( \exp\left(-\frac{mgh}{kT}\right) \)
(e) \( \exp\left(-\frac{2mgh}{kT}\right) \)

[12] A mixture of two gases will “diffuse” through a filter at rates proportional to their rms speeds. What is the ratio of diffusion rates for mono-atomic hydrogen and helium, \( \frac{\text{rate}(\text{H})}{\text{rate}(\text{He})} \)? Assume the two gases are in thermal equilibrium with each other.
(a) 1.0
(b) 2.0
(c) 1.1
(d) 0.5
(e) 0.99

[13] Which of the following pair of linear wave equations is satisfied by only left-moving (i.e. towards \(-x\)) small-amplitude transverse displacement waves on a string, given by \( y(x,t) \), when the tension in the string is \( F \) and the mass/length of the string is \( \mu \)?
(a) \( \mu \partial^2 y/\partial t^2 = -F \partial^2 y/\partial x^2 \) and \( \sqrt{\mu} \partial y/\partial t = \sqrt{F} \partial y/\partial x \)
(b) \( \mu \partial^2 y/\partial t^2 = F \partial^2 y/\partial x^2 \) and \( \sqrt{\mu} \partial y/\partial t = -\sqrt{F} \partial y/\partial x \)
(c) \( \mu \partial^2 y/\partial t^2 = F \partial^2 y/\partial x^2 \) and \( \sqrt{\mu} \partial y/\partial t = \sqrt{F} \partial y/\partial x \)
(d) \( \mu \partial^2 y/\partial t^2 = -F \partial^2 y/\partial x^2 \) and \( \sqrt{\mu} \partial y/\partial t = -\sqrt{F} \partial y/\partial x \)
(e) \( F \partial^2 y/\partial t^2 = \mu \partial^2 y/\partial x^2 \) and \( \sqrt{\mu} \partial y/\partial t = -\sqrt{F} \partial y/\partial x \)

[14] What is the speed of sound in air at room temperature?
(a) 340 meter/sec
(b) 1100 feet/sec
(c) 370 yards/sec
(d) 750 miles/hr
(e) Sound doesn’t have a speed, dude!
**PHY206 Formulas:** \( \ln (A/B) = \ln A - \ln B \)

\[
x^2/a^2 + y^2/b^2 = 1 \quad \int \frac{dx}{x} = \ln x, \quad \int dx e^{sx} = \frac{1}{s} e^{sx}, \quad \int dx x e^{-x^2} = -\frac{1}{2} e^{-x^2} \\
L = m \rho x \quad \int_0^{+\infty} ds s e^{-s} = 1 = \int_0^{+\infty} ds s e^{-a s}, \quad \int_{-\infty}^{+\infty} ds e^{-s^2} = \sqrt{\pi} \\
G_{\text{Newton}} = 6.67 \times 10^{-11} \text{Nm}^2/\text{kg}^2 \quad T_{\text{Kelvin}} = T_{\text{Celsius}} + 273.15, \quad 1 \text{ cal} = 4.184 \text{ J} \\
g = 9.8 \text{ m/s}^2 \quad T_{\text{Celsius}} = \frac{9}{5} (T_{\text{Fahrenheit}} - 32) \\
k \equiv k_{\text{Boltzmann}} = 1.38 \times 10^{-23} \text{ J/K} \\
R = 8.31 J/ (K \text{ mole}) \\
N_A = 6.02 \times 10^{23} \text{ particles/mole} \\
STP: 273.15 K and 1 atm = 1.013 \times 10^5 \text{ N/m}^2 \\
\]

\[P + \frac{1}{2} \rho \nu^2 + \rho gh = \text{constant along streamlines (Bernoulli’s equation)} \]

\[
PV = nRT = NkT, \quad \frac{1}{2} m \langle \nu^2 \rangle_{\text{average}} = \frac{3}{2} kT, \quad \langle KE \rangle_{\text{total}} = \frac{3}{2} nRT = U_{\text{ideal mono gas}} \\
\nu_{\text{rms}} = \sqrt{\frac{3kT}{m}}, \quad \nu_{\text{most likely}} = \sqrt{\frac{2kT}{m}}, \quad f(\nu) = 4\pi \nu^2 \left( \frac{m}{2\pi kT} \right)^{3/2} \exp \left( -\frac{\nu^2}{2kT} \right), \\
d^3v = dv_x \ dv_y \ dv_z = v^2 \ dv \ d\phi \ sin \ \theta \ d\theta \\
C_P = C_V + nR, \quad C_V = f \times \frac{N}{2} kT, \quad \gamma \equiv C_P/C_V = \frac{f + 2}{f}, \quad f_{\text{mono}} = 3, \quad f_d = 5 \\
\Delta Q_{\text{phase change}} = M L_{\text{heat}}, \quad \Delta Q = C \Delta T, \quad \Delta W = P \Delta V, \quad \Delta Q = \Delta U + \Delta W \\
PV^\gamma \text{ and } TV^{\gamma - 1} \text{ are ideal gas adiabatic invariants}, \quad \frac{W_{\text{ideal gas adiabatic expansion}}}{\gamma - 1} = \frac{(PV)_{\text{initial}} - (PV)_{\text{final}}}{\gamma - 1} \\
\]

\[
dS = \frac{(dQ)_{\text{irreversible}}}{T}, \quad dS_{\text{total irreversible}} > 0, \quad dS_{\text{ideal gas}} = C_V \left( \frac{dT}{T} \right) + nR \left( \frac{dV}{V} \right) \\
S = -nk \ln (\text{Probability}) + \text{constant} \\
\left| \frac{\Delta Q_{\text{out}}}{\Delta Q_{\text{in}}} \right|_{\text{Carnot}} = \frac{T_{\text{out}}}{T_{\text{in}}}, \quad \varepsilon = 1 - \left| \frac{\Delta Q_{\text{out}}}{\Delta Q_{\text{in}}} \right| \\
\frac{dL}{L} = \alpha dT, \quad \frac{dQ}{dt} = -\kappa_{\text{thermal}} \frac{dT}{dx} \\
\lambda_{\text{black-body-at-max}} = \frac{2.898 \text{ mm K}}{T}, \quad P_{\text{Stefan power law}} = \varepsilon_{\text{emissivity}} \sigma AT^4, \quad \sigma = 5.67 \times 10^{-8} \text{ Watts/m}^2\text{K}^4 \\
\]

\[
v = \sqrt{\frac{F}{\mu}} \text{ for waves on a spring (or string) under tension } F \text{ and with linear mass density } \mu \\
v_{\text{adiabatic}} = \sqrt{\frac{\gamma kT}{m}}, \quad \frac{\partial^2 f}{\partial t^2}(x,t) = v_{\text{adiabatic}}^2 \frac{\partial^2 f}{\partial x^2}(x,t) \\
\]