Please attempt ALL problems.

This is a “closed-book” examination.

You may NOT use your notes, nor any textbooks.

You must work independently. NO collaboration is allowed.

You may ask Dr. Curtright questions, however, especially if you think something is unclearly stated.

GOOD LUCK !!

“On my honor, I have neither received nor given aid on this exam.”

Name: ________________________________________________

Signature: _____________________________________________

ID #: _______________________________________________
Exam total score (102=6x17 possible):


Please Note: In all of the following, **bold letters** denote vectors. Unit vectors along the $x$, $y$, and $z$ axes are denoted by $\mathbf{e}_x$, $\mathbf{e}_y$, and $\mathbf{e}_z$, respectively. A unit vector pointing radially away from the origin is denoted by $\mathbf{e}_r$. 
Multiple Choice Questions

Circle or underline the correct answer.

*Sometimes by accident or by design, none of the given answers may be correct. If you believe that to be the case, you should write-in what you believe to be the correct answer.*

[1] In the Figure below, the current in the long, straight wire is $I_1$ and the wire lies in the plane of the rectangular loop which carries current $I_2$. What is the magnitude and direction of the net force exerted on the loop by the magnetic field created by the wire?

\[
\begin{align*}
(1) \quad & \mu_0 I_1 I_2 \left( \frac{1}{2\pi c} - \frac{1}{2\pi (a+c)} \right) \hat{e}_{left} \\
(2) \quad & \mu_0 I_1 I_2 \left( \frac{1}{2\pi c} + \frac{1}{2\pi (a+c)} \right) \hat{e}_{left} \\
(3) \quad & \mu_0 I_1 I_2 \left( \frac{1}{2\pi c} - \frac{1}{2\pi (a+c)} \right) \hat{e}_{right} \\
(4) \quad & \mu_0 I_1 I_2 \left( \frac{1}{2\pi c} - \frac{1}{2\pi (a+c)} \right) \hat{e}_{left} \\
(5) \quad & \mu_0 I_1 I_2 \left( \frac{1}{2\pi c} + \frac{1}{2\pi (a+c)} \right) \hat{e}_{right}
\end{align*}
\]

[2] An insulated conductor consists of a circular loop of radius $R$ and two straight long sections, as in the Figure below. The wire lies in the plane of the paper and carries a current $I$. What is the magnetic field at the center of the loop?

\[
\begin{align*}
(1) \quad & \frac{\mu_0 I}{4\pi R} (2 + 2\pi) \hat{e}_{into\ page} \\
(2) \quad & \frac{\mu_0 I}{4\pi R} (2 - 2\pi) \hat{e}_{into\ page} \\
(3) \quad & \frac{\mu_0 I}{4\pi R} \hat{e}_{into\ page} \\
(4) \quad & \frac{\mu_0 I}{4\pi R} (2 - 2\pi) \hat{e}_{out\ of\ page} \\
(5) \quad & \frac{\mu_0 I}{4\pi R} (2 + 2\pi) \hat{e}_{out\ of\ page}
\end{align*}
\]
A wide copper strip of width $W$ is bent into a piece of slender tube of radius $R$ with two plane extensions that almost touch, as shown in the Figure below. You should assume that $R \ll W$ in the following questions. A current $I$ flows through the strip, distributed uniformly over its width. In this way a “one-turn solenoid” has been formed.

[3] What is the magnitude of the magnetic field inside the current tube, near the middle of the tube?

- (1) $B = \mu_0 I/W$
- (2) $B = \mu_0 I/R$
- (3) $B = \mu_0 I R$
- (4) $B = \mu_0 I W$
- (5) $B = 0$

[4] What is the magnitude of the magnetic field outside of the current tube, again near the middle of the tube?

- (1) $B = 0$
- (2) $B = \mu_0 I/W$
- (3) $B = \mu_0 I R$
- (4) $B = \mu_0 I W$
- (5) $B = \mu_0 I/R$

[5] How much energy per unit volume, $u_B$, is stored in the tube, near the middle?

- (1) $u_B = \mu_0 I^2 / (2W^2)$
- (2) $u_B = \mu_0 I^2 / (2R^2)$
- (3) $u_B = \mu_0 I^2 R^2/2$
- (4) $u_B = \mu_0 I^2 W^2/2$
- (5) $u_B = 0$

[6] What is the magnetic flux through the tube?

- (1) $\Phi_B = \pi \mu_0 I R^2/W$
- (2) $\Phi_B = \pi \mu_0 I R^2 W$
- (3) $\Phi_B = \pi \mu_0 I R$
- (4) $\Phi_B = \pi \mu_0 I R^3$
- (5) $\Phi_B = 0$
[7] Two single-turn circular loops of thin wire have radii $R$ and $r$, with $R \gg r$. The loops lie in the same plane and are concentric. Current $I$ flows around the larger loop. What is the resulting magnetic flux $\Phi_B$ through the smaller loop?

$(1) \ \mu_0 \pi R^2 / 2R \quad (2) \ \mu_0 \pi R^2 / 2r \quad (3) \ 2RI / \mu_0 \pi r^2 \quad (4) \ 2rI / \mu_0 \pi R^2 \quad (5) \ 0$

[8] A circular loop of area 1.0 m² is rotating about one of its diameters at $\omega = 100$ radians/sec with the axis of rotation perpendicular to a 0.10 T magnetic field. If there are 100 turns of insulated conducting wire around the loop, what is the maximum voltage induced in the wire?

$(1) \ 1.0 \ kV \quad (2) \ 7.5 \ kV \quad (3) \ 20 \ V \quad (4) \ 7.5 \ V \quad (5) \ 10 \ kV$

[9] A 1.0 mH inductor has a current going through it as given by $I = I_{max} \sin(\omega t)$ with $I_{max} = 10.00$ A and $\omega = 100$ radians/sec. What is the voltage drop (in the direction of the current, in volts) in the inductor as a function of time?

$(1) \ 1.0 \cos(100t) \quad (2) \ 1.0 \sin(100t) \quad (3) \ There\ is\ none.\ \quad (4) \ 5.0 \cos(100t) \quad (5) \ 5.0 \sin(100t)$

[10] A series “RL” circuit and a series “RC” circuit have the same time constant. If the “RL” circuit has resistance $R$, and the “RC” circuit has resistance $4R$, what is the value of $R$ in terms of $L$ and $C$?

$(1) \ R = \frac{1}{4} \sqrt{L/C} \quad (2) \ R = \sqrt{L/C} \quad (3) \ R = \frac{1}{4} \sqrt{L/C} \quad (4) \ R = \sqrt{LC} \quad (5) \ R = LC$

[11] A capacitor consisting of two very close parallel plates is initially uncharged. A current $I(t) = I_{max} \sin(\omega t)$ is switched on at time $t = 0$ and begins flowing onto the lower plate of the capacitor. According to Gauss’s law, what is the upward electric field between the capacitor plates as a function of time? Let $A$ represent the area of each plate.

$(1) \ E = \frac{I_{max} (1 - \cos(\omega t))}{\varepsilon_0 \omega A} \quad (2) \ E = \frac{I_{max} \sin(\omega t)}{\varepsilon_0 \omega A} \quad (3) \ E = \frac{I_{max} \varepsilon_0 \omega A \sin(\omega t)}{\varepsilon_0 \omega A}$

$(4) \ E = \frac{I_{max} \cos(\omega t)}{\varepsilon_0 \omega A} \quad (5) \ E = 0$

[12] The capacitor plates in the previous problem are separated by a distance $d$, with the same current flowing into it as before. What is the voltage difference at time $t$ across the plates of this capacitor?

$(1) \ V = dI_{max} (1 - \cos(\omega t)) / \varepsilon_0 \omega A \quad (2) \ V = dI_{max} \cos(\omega t) / \varepsilon_0 \omega A \quad (3) \ V = dI_{max} \varepsilon_0 \omega A \sin(\omega t)$

$(4) \ V = dI_{max} \cos(\omega t) / \varepsilon_0 \omega A \quad (5) \ V = 0$
A long, straight copper wire lies along the z axis and has a circular cross section of radius R. In addition, there is a circular hole of radius a running the length of the copper wire, parallel to the z axis. The center of the hole is located at \( x = b \), with \( a + b < R \). A cross section of the wire is shown in the Figure below.

Suppose a total current \( I \) flows uniformly through the copper wire, in the \(+e_z\) direction (out of the page in the Figure).

[13] The current density inside the wire is given by \( \mathbf{J} = J e_z \). What is \( J \)?

\[
\begin{align*}
(1) & \quad J = \frac{I}{\pi (R^2 - b^2)} \\
(2) & \quad J = \frac{I}{\pi R^2} \\
(3) & \quad J = \frac{I}{\pi a^2} \quad \text{None correct! Correct answer is} \quad J = \frac{I}{\pi (R^2 - a^2)} \\
(4) & \quad J = \frac{I}{\pi (R^2 + b^2)} \\
(5) & \quad J = \frac{I}{2\pi R}
\end{align*}
\]

[14] Suppose the hole is not there (as would be the case if \( a = 0 \)), but the same current density as before flows inside the wire. What would the magnetic field be on the \( x \) axis, at a distance \( r \) from the origin, inside the wire with \( b < r < a + b \)? (Hint: Ampere’s Law)

\[
\begin{align*}
(1) & \quad \mathbf{B} = \frac{1}{2} \mu_0 J r e_y \\
(2) & \quad \mathbf{B} = \frac{1}{2} \mu_0 J r e_x \\
(3) & \quad \mathbf{B} = \frac{1}{2r} \mu_0 J R^2 e_y \\
(4) & \quad \mathbf{B} = \frac{1}{2r} \mu_0 J R^2 e_x \\
(5) & \quad \mathbf{B} = 0
\end{align*}
\]
[15] Now suppose the wire of radius $R$ is not there, and in place of the hole there is a smaller circular wire of radius $a$, with center at $x = b$, carrying the same current density $J$, but into the page. What would the magnetic field be on the $x$ axis, at a distance $b < r < a + b$ from the origin, inside this smaller wire?

\[
\begin{align*}
1. \quad B &= \frac{1}{2} \mu_0 J (b - r) e_y \\
2. \quad B &= \frac{1}{2} \mu_0 J (b - r) e_x \\
3. \quad B &= \frac{1}{2} \left( \frac{1}{r-b} \right) \mu_0 J a^2 e_y \\
4. \quad B &= \frac{1}{2} \left( \frac{1}{r-b} \right) \mu_0 J a^2 e_x \\
5. \quad B &= 0
\end{align*}
\]

[16] By combining your answers to the previous two problems, you can determine the magnetic field for the original wire with the hole as shown in the above Figure. What would the magnetic field be on the $x$ axis, at a distance $b < r < a + b$ from the origin, inside the hole in the wire?

\[
\begin{align*}
1. \quad B &= \frac{1}{2} \mu_0 J b e_y \\
2. \quad B &= \frac{1}{2} \mu_0 J (2r - b) e_y \\
3. \quad B &= \left( \frac{1}{2r} \right) \mu_0 J R^2 + \frac{1}{2} \mu_0 J a^2 e_y \\
4. \quad B &= \left( \frac{1}{2r} \right) \mu_0 J R^2 + \frac{1}{2} \mu_0 J a^2 e_x \\
5. \quad B &= \frac{1}{2r} \mu_0 J R^2 e_y
\end{align*}
\]

[17] What would the magnetic field be anywhere inside the hole in the wire shown in the Figure, at a location $(x, y)$ and at a distance $r = \sqrt{x^2 + y^2}$ from the origin?

\[
\begin{align*}
1. \quad B &= \frac{1}{2} \mu_0 J b e_y \\
2. \quad B &= \frac{1}{2} \mu_0 J (x e_y - ye_x) \\
3. \quad B &= \left( \frac{1}{2r} \right) \mu_0 J R^2 + \frac{1}{2} \mu_0 J a^2 e_y \\
4. \quad B &= \left( \frac{1}{2r} \right) \mu_0 J R^2 + \frac{1}{2} \mu_0 J a^2 e_x \\
5. \quad B &= \frac{1}{2r^2} \mu_0 J R^2 (x e_y - ye_x)
\end{align*}
\]