# Vector Goldstone Boson and Lorentz Invariance*

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**Abstract**

Spontanous symmetry breaking usually gives spin 0 Goldstone bosons for the case of internal symmetries and spin 1/2 fermions for the supersymmetry. The spontaneous breaking of higher dimensional Lorentz symmetry can give vector Goldstone boson in 4-dimension.

## 1 Introduction

Spontaneous symmetry breaking has produced many interesting phenomena in many different areas of physics. The celebrated Goldstone theorem ([1]) implies the presence of zero energy excitation for the spontaneous breaking of a continuous symmetry. In the context of relativistic field theory, this gives rise to a massless particle, the Goldstone particle. For the case of internal symmetries, the spontaneous symmetry breaking gives massless spin zero mesons, the Goldtone bosons, while the spontaneous breaking of supersymmetry yields massless spin 1/2 fermions, the Goldstinos. On the other hand, to have Goldstone boson with spin 1 seems to require breaking of Lorentz symmetry. In this note, we discuss the relation between the vector

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Goldstone boson and Lorentz invariance. It turns out that in theories with extra dimensions ([6]) it is possible to have vector Goldstone boson without breaking the 4-dimensional Lorentz symmetry ([5]) .

## 2 Spin 0 and 1/2 Goldtone Particles

We first illustrate the Goldstone theorem ([**?**]) to set the framework for the discussion. Suppose the Lagrangian density is invariant under a continuous transformation which gives rise to a conserved current,

$$\partial^{\mu} J_{\mu}(x) = 0.$$

This implies a conserved charge which is the generator of the symmetry transformation,

$$Q = \int J_0(x)\, d^3x, \qquad \frac{dQ}{dt} = 0$$

Suppose that there are two local operators $A$ and $B$, which are related by the generator of the symmetry transformation $Q$,

$$[Q,\ A] = B. \tag{1}$$

If for some dynamical reasons, the operator $B$ has non-zero vacuum expectation value (VEV),

$$\langle 0 | B | 0 \rangle \neq 0, \tag{2}$$

we say that the symmetry is broken spontaneously. It follows from Eq(2) that

$$Q|0 > \neq 0 \tag{3}$$

i. e. the vacuum is no longer invariant under the symmetry transformation. The Goldstone theorem then states that there exists a state $|g\left(\overrightarrow{p}\right) >$ with the property that its energy

$$E_g\left(\overrightarrow{p}\right) \rightarrow 0, \qquad \text{as} \qquad \overrightarrow{p} \rightarrow 0.$$

Here $\overrightarrow{p}$ is the momentum of the particle. From the relativistic energy momentum relation, $E = \sqrt{\overrightarrow{p}^2 + m^2}$ this implies the existence of massless particle. Note that the Goldstone state $|g\left(\overrightarrow{p}\right) >$ couples directly to the operators, $A$ and $J_\mu$,

$$\left\langle 0\left|J_\mu\right| g\left(\overrightarrow{p}\right)\right\rangle \neq 0, \qquad \left\langle 0\left|A\right| g\left(\overrightarrow{p}\right)\right\rangle \neq 0.$$

This means that the Goldstone state $|g\left(\overrightarrow{p}\right) >$ is also the quantum of the local operator $A$. Note that the symmetry breaking condition in Eq (2) implies that whatever the quantum number carried by the operator $B$ will not be conserved by the vacuum.

A simple example of spin 0 Goldstone boson is provided by the spontaneous breaking of the chiral symmetry in the low energy hadron physics ([4]). Here chiral charges $Q_5^a$ transform pion fields $\pi^b$ into scalar $\sigma$ field,

$$\left[Q_5^a,\ \pi^b\right] = \delta_{ab}\sigma. \tag{4}$$

Suppose the effective interaction between $\pi$ and $\sigma$ is of the form,

$$V_{eff} = -\frac{\mu^2}{2}\left(\overrightarrow{\pi}^2 + \sigma^2\right) + \frac{\lambda}{4}\left(\overrightarrow{\pi}^2 + \sigma^2\right)^2 \tag{5}$$

This is invariant under chiral $SU\left(2\right)_L \times SU\left(2\right)_R$ transformations. The minimization of $V_{\ eff}$ leads to non-zero VEV for $\sigma$,

$$\left\langle 0\left|\sigma\right| 0\right\rangle = \sqrt{\frac{\mu^2}{\lambda}} \neq 0 \tag{6}$$

In this case, the pion fields $\overrightarrow{\pi}$ , which are partners of $\sigma$ under the chiral transformation, Eq (4), are the massless spin zero Goldstone bosons. From the point of view of more fundamental theory QCD, $\sigma$ field is just the quark bilinear $\bar{q}q$ and Eq (6) is equivalent to

$$\left\langle 0\left|\bar{q}q\right| 0\right\rangle \neq 0$$

and the Goldstone bosons are bound states,

$$\pi^a \tilde{} \bar{q}\gamma_5\tau^a q$$

For the case of spontaneous breaking of supersymmetry, there are two possibilities. One is generated by the chiral superfield where we have the anti-commutation relation,

$$\{Q_\xi, \psi\} = \sqrt{2}F.$$

Here $Q_\xi$ is the supercharge which transform fermions into bosons and vice versa, $\psi$ is the fermion component and $F$ the scalar auxiliary component of the chiral superfield. If $F$ develops non-zero VEV ([8]),

$$\langle 0 \,|F|\, 0\rangle \neq 0$$

then fermion $\psi$, becomes a Goldstone particle ( Goldstino) and $\psi$ is also a partner of a massive physical scalar field. In the case of spontaneous breaking by a vector superfield, the anti-commutatuion relation of interest is,

$$\{Q_\xi, \lambda\} = \sqrt{2}D$$

where $\lambda$ is the fermion component and $D$ is the auxiliary scalar component of the vector superfield. When $D$ develops VEV ([7]),

$$\langle 0 \,|D|\, 0\rangle \neq 0$$

then $\lambda$ is the Goldstone fermion and is a partner of a massive physical vector particle.

## 3 Vector Goldstone Boson

In all the cases discussed above, the fields which develops non-zero VEV all have spin 0 and the Goldstone particles are partners of these fields under the symmetry transformations. In the usual 4-dimensional field theory, the vector field is not related to any scalar field. The only way to get vector Goldstone boson is to break the Lorentz invariance spontaneoulsy, For example, in analogy with Eq (1), suppose there are two vector fields which are related by some symmetry transformation,

$$[Q,\, A^{\mu}] = B^{\mu}$$

If $B^{\mu}$ develops VEV,

$$\langle 0 \left| B^{\mu} \right| 0 \rangle \neq 0 \tag{7}$$

then the quantum of $A^{\mu}$ field, will be a spin 1 massless Goldstone particle. Clearly, the symmetry breaking condition in Eq (7) breaks the Lorentz invariance.. Even though there are stringent experimental limits on the possible violation of Lorentz symmetry, recently there have been renew interest in studying this issue. However, it has been pointed out ([5]) that in theories with extra dimensions, it is possible to have vector Goldstone boson without breaking the Lorentz symmetry in 4-dimension. As an example ([5]), consider a vector field in 5-dimensional theory, $\phi_A$, $A = 0, 1, 2, 3, 4$. In analogy to Eq (5), we can write down an effective interaction of the form,

$$V(\phi) = \frac{\mu^2}{2}\left(\phi_A \phi^A\right) + \frac{\lambda}{4}\left(\phi_A \phi^A\right)^2 .$$

For the case, $\mu^2 > 0$, we can choose

$$\langle 0 \left| \phi_4 \right| 0 \rangle = \sqrt{\frac{\mu^2}{\lambda}} \tag{8}$$

to minimize the potential $V(\phi)$. This will break the Lorentz symmetry in 5-dimension, $SO(4,1)$ to that of 4-dimension, $SO(3,1)$. As a consequence, the 4-dimensional vector fields $\phi_{\mu}$, $\mu = 0, 1, 2, 3$ which are partners of $\phi_4$, are massless, the vector Goldstone boson. Note that symmetry breaking condition in Eq (8) can come from fermion condensate,

$$\left\langle 0 \left| \bar{\psi} \gamma_A \psi \right| 0 \right\rangle = v \delta_{A4}$$

In this case the vector Goldstone boson correspond to the composite fields $\bar{\psi}\gamma_\mu\psi$, $\mu = 0, 1, 2, 3$.

## 4 Goldstone Photon

It was originally explored by Bjorken forty years ago ([9]) the possibility that the photon is massless because of spontaneous symmetry breaking rather gauge invariance. This idea has been revisited more recently by Bjorken and several other authors ([10]). Even though this idea does not seem to be very attractive in view of the spectacular success of gauge theories in recent years, it is of interest to study whether this idea is phenomenological viable. As we have mentioned before, in 4-dimension to have photon as vector Goldstone boson requires the breaking of Lorentz invariance. We will now discuss a simple version of this scheme to illustrate the idea. The starting point is to write down a self-interacting fermion field theory of the form,

$$L = \bar{\psi}\left(i\gamma^\mu\partial_\mu - m\right)\psi + \sum_{n=1}^{\infty}\lambda_{2n}\left(\bar{\psi}\gamma_\mu\psi\right)^{2n} \tag{9}$$

where $\lambda_{2n}$ 's are coupling constants. This resembles the Nambu-Jona-Lasino type of theory ([2]) for the spontaneous symmetry breaking. One can introduce an auxiliary field $A^\mu$ to rewrite the Lagrangian as,

$$L = \bar{\psi}\left(i\gamma^\mu\partial_\mu - m\right)\psi + eA^\mu\bar{\psi}\gamma_\mu\psi - V\left(A^\mu A_\mu\right)$$

where the effective potential $V$ is

$$V\left(A^2\right) = \Lambda^4\sum_{n=1}^{\infty}V_n\left(\frac{A^2}{\Lambda^2}\right)^n \qquad \text{with} \quad A^2 = A^\mu A_\mu$$

Here $V_n$'s are dimensionless constants related to $\lambda_{2n}$ in Eq (9) and $\Lambda$ is some parameter with dimension of mass. This potential can generate a non-zero VEV of the form,

$$\langle 0\left|A_\mu\right|0\rangle = c\Lambda n_\mu,$$

Here $c$ is some dimensionless constant and $n_\mu$ a space-like unit vector. This breaks the Lorentz symmetry and give 3 massless Goldstone modes in which two of them are the transverse photons and the other one is a time-like mode. If $\Lambda$ is very large, at energies small compared with $\Lambda$ we can have approximate Lorentz symmetry and any deviation can be suppressed by making $\Lambda$ very large enough.

The possible violation of Lorentz invariance has also been investigated in the studies of string theory and gravitational interaction ([12]). The possibility that the graviton is also a Goldstone boson has been discussed in the literature([11]).

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