HIGH SPIN FIELDS

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This is a brief qualitative review of some recent attempts to
formulate local field theories of fundamental, structureless
particles with arbitrary spin. Despite these attempts there are no
known consistent theories, based on a local lagrangian, describing
higher (s>2) spin particles which interact either mutually, with
Einstein gravity, or with lower spins (i.e. s<2). We outline here
the technical reasons why such local formulations are nonexistent
and we indicate areas possibly deserving further study.

We are motivated to study higher spin field theories because of
their intrinsic mathematical interest (as was Dirac\(^1\)), and because
linear supermultiplets with manifest invariance groups large enough
to contain SU(3)\(_c\) x (SU(2)xU(1))\(_{EW}\) require spins s\(\geq5/2\). Conceivably,
such high spin linear representations of extended supersymmetries
may not be present in Nature, even if supersymmetry is. Neverthe-
less, the higher spin formalism deserves consideration for possible
physical applications, even if such applications are only to approx-
imate higher spin composite particles, or to describe higher spin
auxiliary fields in extended supergravities.

Historically, Fierz and Pauli\(^2\) first emphasized the technical
benefits of describing relativistic particles with any spin by using
a local lagrangian framework. They discussed all spins s<5/2 in
detail, including all gauge invariances present for s<5/2 when m=0.
They also pointed out that higher spins require a large number of
auxiliary fields, for massive particles, in order to obtain wave
equations defining a unique on-shell mass and spin. The necessary
structure involving such auxiliary fields delayed the explicit
construction of even free field lagrangians for arbitrary spin and
nonzero mass for 35 years\(^3\). Only within the last two years was it
finally realized that the m=0 case is conceptually quite simple for
arbitrary spin\(^4\)-\(^8\).

All higher spin massless fields may be viewed as gauge fields.
For example, consider a free spin 5/2 particle described by the
symmetric spinor-tensor \(\psi_{\mu\nu}\). Under the Lorentz group, \(\psi_{\mu\nu}\) trans-
forms as \((3/2 + 1/2, 1) + (1/2, 0)\), and carries spins 5/2 + 2(3/2)
+ 3(1/2). By analogy with known lower spin cases, one expects the
gauge invariance under

\[
\delta\psi_{\mu\nu} = \partial_\mu \eta_{\nu} + \partial_\nu \eta_{\mu}
\]  

(1)

to reduce the number of physical propagating massless modes to only
helicities +5/2 and -5/2. This is not quite correct. There is no
local lagrangian of the form \(\bar{\psi}\partial\psi\) which yields an invariant action

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under (1) unless constraints are imposed on $\eta_\mu(x)$, the spinor-vector
gauge parameter. A sufficient and simple constraint is
\[ \gamma^\mu \eta_\mu(x) = 0, \] (2)
which leads to the unique gauge invariant action
\[ A_{5/2} = \int dx (\overline{\psi}_{\mu
u} \gamma^\mu \gamma^\nu \psi_{\mu
u} + 2 \overline{\psi}_\mu \gamma^\mu \psi_{\mu} - \frac{1}{2} \overline{\psi} \gamma^a \psi - 4 \overline{\psi}_\mu \gamma^\nu \psi_{\mu
u} + 2 \overline{\psi}_a \gamma^\mu \psi^a) \] (3)
where $\psi_\gamma = \gamma^\nu \psi_{\nu\mu}$, $\psi = \psi_{\mu}$, and $\psi_{\mu\nu}$ is Majorana for simplicity.

The restricted gauge invariance of (1) and (2) is still
sufficiently strong to eliminate all spurious intermediate helicity
modes. This was established by explicit examination of propagator
pole residues\(^6,\)\(^7\), by reducing the action to (Hamiltonian) canonical
form\(^8\), and by explicitly constructing supermultiplets combining such
$s=5/2$ gauge fields with known lower spins\(^7\).

These same considerations apply to all higher spins described in
terms of totally symmetric tensors and spinor-tensors, with addition-
al constraints on the fields for $s>3$. Namely,
\[ \gamma^\mu \gamma^\nu \gamma^\lambda \psi_{\mu\nu\lambda\cdots\rho} = 0 \text{ for } s>7/2 \]
\[ \phi^\mu_{\mu\nu\lambda\cdots\rho} = 0 \text{ for } s>4. \] (4)

The gauge transformations are the now obvious extensions
\[ \delta \phi_{\mu\nu\lambda\cdots\rho} = \partial (\mu \xi_{\nu\lambda\cdots\rho}) \text{ with } \xi^\nu_{\nu\rho\cdots\rho} = 0 \]
\[ \delta \psi_{\mu\nu\lambda\cdots\rho} = \partial (\mu \eta_{\nu\lambda\cdots\rho}) \text{ with } \gamma^\nu_{\nu\rho\cdots\rho} = 0 \] (5)

Unique actions follow from postulating invariance under (5) with (4).

Three other formal developments for free fields with arbitrary
spin are noteworthy. The BRS formulations of the gauge invariances
above are known\(^6,\)\(^9\) and the corresponding covariant quantization of
the free fields has been carried out. A more "geometrical" founda-
tion for free higher spins has been initiated through the construc-
tion of generalized Christoffel symbols\(^10\) (which include gauge
invariant objects) formed by taking higher derivatives of the fields.
Finally, "vierbein" (i.e. non-symmetric tensor) formulations have
been given which are equivalent to the totally symmetric tensor
formalism\(^11\). This last approach avoids the above constraint dis-
continuity at $s=7/2$ (cf. eqn. (4)).

Free higher spin fields are thus pleasingly described as local
gauge fields. Unfortunately, the introduction of local interactions
leads to difficulties in at least three areas: 1) Higher spin
charges; 2) Consistency conditions; 3) Causality violations. We
briefly summarize results on these difficulties.

First, there are no higher spin (>1) charges\(^12\) to which higher
spin (>2) massless fields could couple at zero momentum. Thus all such fields must decouple as q→0. (Perhaps more dramatic, a related analysis shows m=0 Born amplitudes for s=5/2 coupled to gravity must vanish identically\textsuperscript{13}.)

Second, the equations of motion for massless higher spin fields are inconsistent when minimal coupling to gravity is attempted\textsuperscript{14} unless unrealistic conditions are imposed on the curvature of spacetime (e.g. Weyl flat spaces). This may be seen by differentiating the field equations. These inconsistencies are directly related to the loss of gauge invariance for the minimally coupled action. More physically, unless the necessary curvature conditions are met, energy-momentum does not seem to be conserved\textsuperscript{15}. For the massless case, nonminimal couplings apparently do not alleviate these inconsistencies.

Third, if masses are introduced, the wave equations can be made consistent, but a new problem arises: Acausal propagation\textsuperscript{16}. In general, spacelike characteristic surfaces pass through points with nonvanishing curvature, even in the weak field limit.

We now indicate some open questions inspired by the above difficulties. a) What constraints apply to m>0, s>2 Born amplitudes in the presence of gravitational interactions? b) Can both local mass terms and local nonminimal couplings be introduced to yield consistent, causal higher spin theories in such a way that the m→0 limit is unavoidably singular? c) Interactions with gravity aside, can higher spins be consistently and causally self-coupled?

One naturally guesses analogies with Yang-Mills theory or Einstein gravity as self-coupling candidates, but a more promising approach for answering the third question may be by imitating the nonlinear Born-Infeld model of self-coupled electrodynamics, where

\[ L = \left[ 1 + K F_{\mu \nu} F^{\mu \nu} - \frac{1}{3} K^2 (\ast F_{\mu \nu} F^{\mu \nu})^2 \right]^{1/2} \]

For higher spins, a nonlinear lagrangian might be similarly composed of gauge invariant generalized Christoffel symbols. (Recent work suggests that supersymmetry may be essential in constructing such nonlinear theories\textsuperscript{17}.)

The third question above also brings up the crucial notion of "manifestly chargeless" sources, which vanish identically as q→0, as may indeed be found in the Born-Infeld model. Such sources appear natural from another point of view: "Massive dual formulations" for higher spin fields\textsuperscript{18}. Massive dual fields describe spins when m>0, but collapse to give no propagating physical degrees of freedom when m=0 (except for the special cases s=1 and s=3/2). Thus the massless limit is quite singular, even for the free field case. It is an open question, related to b) above, whether such massive higher spin dual fields can interact locally in a fully consistent, causal manner in the presence of gravitation.

Finally, we note the allowed possibility that higher spins can be made to interact only if all such spins are present (cf. conventional dual models). A possible novel approach here is based on representations of the de Sitter group\textsuperscript{19}.

In summary, although consistent local interacting field theories involving higher spin fields are not known, the kinematic foundations upon which such models must be constructed have been considerably
clarified in the last two years by extending the concept of gauge invariance to include all spins. Massless fields with arbitrary spin appear to have insurmountable difficulties when coupled individually to gravity. Massive fields have not been similarly investigated to the point of an impasse, and in our view represent the most promising possibilities for local higher spin field theories with gravitational coupling.

Coincident with many of the higher spin investigations described above has been the discovery that the N=8 supergravity theory has a nonlinearly realized "local" SU(8) invariance\(^{20}\) which may permit supersymmetric theories to incorporate SU(3)\(C\times (SU(2)\times U(1))\)\(_{\text{ew}}\) without the need for fundamental higher spin (>2) fields. This is still conjecture. There is a distinct possibility, however, that some higher spin formalism may play a useful role in the phenomenological application\(^{21}\) of the N=8 theory.

REFERENCES