Nonabelian D-branes, Open Strings and Gauge Theory

(arXiv:0809.1085)

Charles B. Thorn

University of Florida

Miami 2008

Conceptual Basis for Field/String Duality

Open String Trees $\Rightarrow$ All String Tree and Loop Diagrams ($\alpha' > 0$)

Open String Trees $\underset{\alpha' \to 0}{\sim}$ QFT Trees

Practical Basis for Field/String Duality:

’t Hooft’s $N \to \infty \Rightarrow$

$$\sum(\text{Planar Open String Loops}) \equiv \sum(\text{Closed String Trees})$$

Left Side $\underset{\alpha' \to 0}{\Longrightarrow}$ $N = \infty$ Gauge Theory

Right Side $\underset{\substack{\alpha' \to 0 \\ g^2 N \to \infty}}{\Longrightarrow}$ Classical gravity

If $g^2 N = O(1)$, right side stays stringy as $\alpha' \to 0$.

String dual for given QFT

First "lift" QFT trees to consistent open string trees

E.g., the AdS/CFT Paradigm:

Lift $\mathcal{N} = 4$ Yang-Mills to NSR/GSO Open String

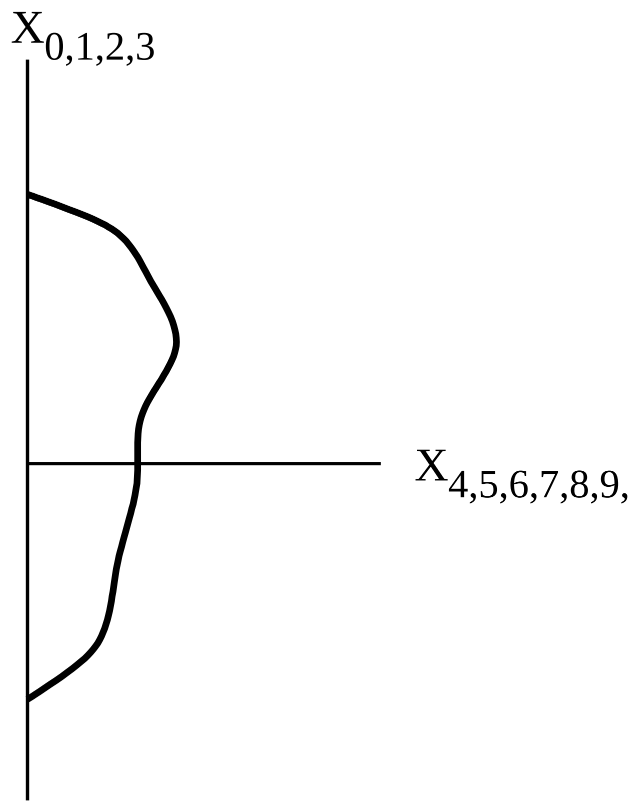
What about pure Yang-Mills?

Open String with Chan-Paton Factors $\underset{\alpha' \to 0}{\Longrightarrow}$ Yang-Mills Theory

10 D NSR Open String/GSO $\underset{\alpha' \to 0}{\Longrightarrow}$ $\mathcal{N} = 1$ 10D SUSY YM

Left side a regulated version of right side

D3-branes and 4D QFT



$$x^M(\sigma,\tau):\begin{cases}\text{Neumann b.c.'s} & M=0,1,2,3\equiv\mu\\ \text{Dirichlet b.c.'s} & M=4,5,6,7,8,9\end{cases}$$

10D translation invariance is broken

String's energy momentum p^μ has 4 space-time components.

Low energy limit is $\mathcal{N} = 4$ 4D SUSY Yang-Mills,
Instead of $\mathcal{N} = 1$ 10D Yang-Mills

Bulk of open string vibrates in all 10 space-time dimensions.

Massless states of NSR open string, 4D P.O.V.:

- an adjoint vector: vibrations $\parallel$ D3-branes,
- 6 adjoint scalars: vibrations $\perp$ D3-branes,
- 4 Majorana fermions.

Field content of $\mathcal{N} = 4$ Yang-Mills

AdS/CFT

$\sum$(planar open string loops) $\Rightarrow \sum$(closed string trees),
Closed strings disappearing into vacuum.

$$\begin{matrix} \alpha' \to 0 \\ \lambda \to \infty \end{matrix} \Rightarrow \text{classical supergravity, sourced by D3-branes}$$

Here 't Hooft coupling: $\lambda \equiv g_s^2 N$.

For $\mathcal{N} = 4$, geometry is $\mathrm{AdS}_5 \times \mathrm{S}^5$.

Search for Open String for Pure 4D Yang-Mills

Delete fermionic states (no R sector)

Even G-parity sector of Neveu-Schwarz (NS+) open string

Simplest choice for YM: NS+ model in 4D

no extra scalars and massive would-be graviton –but ...

Conformal anomaly ⇒ technical complications in closed sector

Keeping 10D ⇒ conformal anomaly cancels.

Usual D3-brane trick $\Rightarrow$ 6 massless scalars in the 4D theory

Produces string dual of Yang-Mills coupled to 6 adjoint scalars (Klebanov, Tseytlin).

Klebanov-Tseytlin argue that R-R closed string stabilizes tachyon

Rest of talk:

Project out the 6 massless scalars by nonabelian D-brane b.c.'s:

Non-abelian D-branes

T-dual D-brane conditions

For a D3-brane at $x^I = 0$:

$$x^I(0,\tau) \quad = \quad x^I(\pi,\tau) = 0$$

T-dual transform $x^I(\sigma,\tau) \to y^I(\sigma,\tau)$:

$$\frac{\partial y^I}{\partial \sigma}(0,\tau) = \frac{\partial y^I}{\partial \sigma}(\pi,\tau) = 0.$$

Then the zero mode of y^I:

$$p_0^I \equiv \int d\sigma \dot{y}^I(\sigma,\tau) = x^I(\pi,\tau) - x^I(0,\tau) = 0$$

SU(2) Invariance

Interpret y^I as a $c = 1$ conformal scalar field,
compactified on a circle: $p_0^I = 2\pi n/R$.

SU(2) symmetry emerges when R is such that
$|0, \pm 2\pi/R\rangle$ are massless.
$(b^I_{-1/2}|0\rangle, |0, \pm 2\pi/R\rangle)$ transform as a vector

Invariance under SU(2) $\Rightarrow n = 0$ **and** projects out $b^I_{-1/2}|0\rangle$

SU(2) invariance for each of 6 extra coordinates,
projects out all massless scalars in open string state space.

SU(2) symmetry, NS Open String

Set $J_3 = p_0^I\sqrt{2\alpha'}$, SU(2) character on state space:

$$\begin{aligned}
X_{\rm NS}(\theta) &\equiv {\rm Tr} w^{L_0}(\pm)^{2R} e^{-i\theta J_3} \\
&= \sum_{k=-\infty}^{\infty} w^{k^2/2} e^{ik\theta} \prod_{r=1/2}^{\infty} (1 \pm w^r) \prod_{n=1}^{\infty} \frac{1}{1-w^n} \\
&= \sum_{k=0}^{\infty} w^{k^2/2} \chi_k(\theta) \prod_{r\neq k+1/2}^{\infty} (1 \pm w^r) \prod_{n\neq 2k+1}^{\infty} \frac{1}{1-w^n} \\
\chi_j(\theta) &\equiv \sum_{m=-j}^{j} e^{im\theta}
\end{aligned}$$

Here χ_j is the character for the spin j irreducible representation of SU(2).

Comments

- Spinor representations of SU(2) are absent (they were in the deleted Ramond sector!),
- Symmetry is O(3).
- Partition function for O(3) invariant subspace:

$$\prod_{r=3/2}^{\infty} (1 \pm w^r) \prod_{n=2}^{\infty} \frac{1}{1 - w^n},$$

- Absence of $r = 1/2$ reflects absence of massless scalar

Vertex operator construction of SU(2) generators

$$J_3 = p_0^I\sqrt{2\alpha'}$$
$$J_\pm = \sqrt{2}\oint \frac{dz}{2\pi i z} H^I(z) : e^{\pm i y^I(z)/\sqrt{2\alpha'}} :$$

$: e^{iy^I(z)/\sqrt{2\alpha'}} :$ is a bosonized fermionic field,
when acting on the states with $p_0^I \in \mathbb{Z}/\sqrt{2\alpha'}$.

$[J_\pm, G_r] = [J_\pm, L_n] = 0$ on this subspace

Also $[J_+, J_-] = 2J_3$.

Manifestly O(3) Invariant Description

Replace each bosonic y^I with a pair of fermion fields H_1^I, H_2^I

Call original $H^I \equiv H_3^I$.

Then H_a transform as a vector under $O(3)$ with generators

$$J^a = \epsilon^{abc} \oint \frac{dz}{2\pi i z} H_b(z) H_c(z).$$

Nonabelian D-brane condition: $J^a|\mathrm{Phys}\rangle = 0$.

Scattering Amplitudes

Trees are subset of 10D NS trees:

Take external strings only in even G-parity states invariant under 6 SU(2)'s associated with the 6 extra dimensions.

Massless scalar states $b^I_{-1/2}|0,p\rangle$, with $I = 4, 5, \ldots, 9$ decouple

One Loop and Closed Strings

Loops require projectors onto SU(2) invariant states.

At one loop simply multiplies partition function by $(1 \mp w^{1/2})^6$
Also loop momentum integral only over p^μ, $\mu = 0, 1, 2, 3$

After Jacobi transformation to cylinder variables $\ln q = 2\pi^2 / \ln w$, these differences produce the extra factors

$$\left[\sqrt{\frac{-\pi}{\ln q}}(1 \mp w^{1/2})\right]^6 = \begin{cases} \left[\int d\mu q^{\mu^2/4} \sin^2 \frac{\mu}{2\sqrt{2}}\right]^6 \\ \left[\int d\mu q^{\mu^2/4} \cos^2 \frac{\mu}{2\sqrt{2}}\right]^6 \end{cases}$$

relative to the usual one loop integrand in $D = 10$.

Closed String Coupling to Nonabelian D3-Branes

Difference from the normal D-brane integrand is factor $\psi_{\pm}^2(\mu^I)$

$$\psi_+(\mu^I) = 2^3 \prod_{I=4}^{9} \sin(\mu^I/2\sqrt{2}), \quad \psi_-(\mu^I) = 2^3 \prod_{I=4}^{9} \cos(\mu^I/2\sqrt{2})$$

The $+$ and $-$ refer to the NS-NS and R-R sectors, respectively, of the type-0 closed string system.

Factor $\psi_{\pm}$ introduces an asymmetry in the coupling of these two sectors compared to the normal D-brane.
NS-NS suppressed relative to R-R at low energy ($\mu \ll 1$)

Type-0 Closed Strings

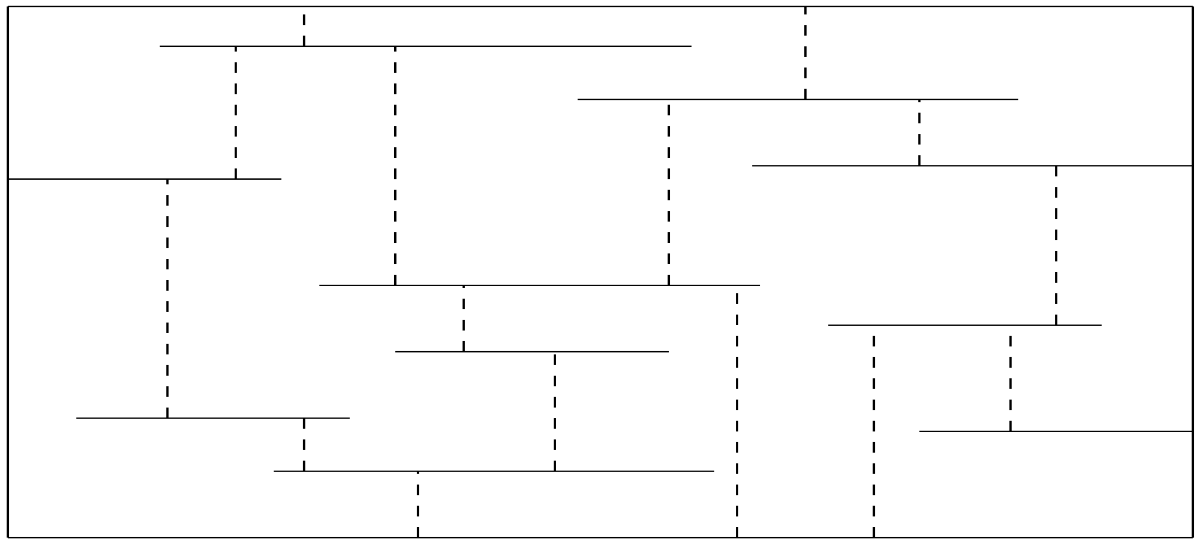
The closed string system that propagates in the 10 dimensional bulk is the critical type-0 closed string, which is well-studied.

The coupling of these closed strings to a nonabelian D3-brane is significantly different from the coupling to the normal (abelian) D3-brane.

Summing Planar Diagrams on the Lightcone

Lightcone path integration description provides one systematic approach to this problem

Potentially very useful for a numerical assault on the problem.



Planar lightcone open string diagram with 7 loops
Dashed lines indicate projector placements

The horizontal dimension is lightcone time $x^+ = (t+z)/\sqrt{2}$ and the vertical dimension is $p^+ = (p^0 + p^z)/\sqrt{2}$.

The diagram describes the evolution in time of a system of open strings, breaking and rejoining as shown by the horizontal lines.

For the critical open string, it is a worldsheet path integral using the lightcone action for the free open string.

$\sum$(planar diagrams)= $\sum$(#, length, location of horizontal lines).

For each beginning and end of a horizontal line there is a factor of string coupling $g\times$(prefactor).

Lightcone Worldsheet

Mark the presence or absence of a horizontal line at any point by an Ising spin variable $S(\sigma, \tau) = \pm 1$. Execute the loop sum as a sum over all spin configurations, with spin dependent terms in the action enforcing the necessary boundary conditions.

It is essential that the path integrand be local. Normal D-brane conditions are applied locally on the worldsheet $x_{\rm ends} = 0$. But this condition looks non-local in T-dual variables $\int d\sigma \dot{y} = 0$, which were used to formulate nonabelian D-brane conditions.

Need to consider them a little more carefully.

O(3) Projectors

Projector for the Ith SU(2):

$$P_I = \int dR e^{i\theta_a J_I^a}, \qquad J_I^a = \int d\sigma \mathcal{J}_I^a(\sigma)$$

dR is the O(3) invariant Haar measure.

The number of such projector $P = \prod_{I=4}^{9} P_I$ insertions changes with the appearance or disappearance of a horizontal line.

Gauging the O(3)'s

Since $P^2 = P$ we can put a projector on each time-slice of each string propagator. Introduce independent R's for each point σ, t:

$$\begin{aligned} P_I &= \prod_t P_I = \int \prod_t dR(t) e^{i\theta_a(t) J_I^a} \\ &= \int \prod_t \prod_\sigma dR(\sigma, t) e^{i \int d\sigma \theta_a(\sigma,t) \mathcal{J}_I^a} \delta(R'(\sigma, t)) \end{aligned}$$

Delete the $\delta(R'(\sigma, t))$ factor whenever σ sits on a horizontal line.

We have gauged each O(3) symmetry on the worldsheet, replacing the factor $e^{-\int F^2/4}$ with $\prod_{\sigma,t} \delta(F(\sigma, \tau))$.

Worldsheet local prescription for inserting projectors.

Summary

- Open String ($\alpha' > 0$) determines/regulates gauge theory
- Open/Closed Duality $\underset{\alpha' \to 0}{\Longrightarrow}$ Field/String Duality
- Open String for 4D Yang-Mills:
  10D NS+ with nonabelian D3-brane b.c.'s on 6 dimensions
- Lightcone path integrals: a non-perturbative formulation for $\sum$(Planar Diagrams)

References

[1] J. Maldacena, Adv. Theor. Math. Phys. **2**, 231 (1998), hep-th/9711200.

[2] G. 't Hooft, *Nucl. Phys.* **B72** (1974) 461.

[3] E. Witten, Adv. Theor. Math. Phys. **2** (1998) 505 [arXiv:hep-th/9803131].

[4] A. Karch and E. Katz, JHEP **0206** (2002) 043 [arXiv:hep-th/0205236]; M. Kruczenski, D. Mateos, R. C. Myers and D. J. Winters, JHEP **0307** (2003) 049 [arXiv:hep-th/0304032]; M. Kruczenski, D. Mateos, R. C. Myers and D. J. Winters, JHEP **0405** (2004) 041 [arXiv:hep-th/0311270]; T. Sakai and S. Sugimoto, Prog. Theor. Phys. **113** (2005) 843 [arXiv:hep-th/0412141].

[5] A. M. Polyakov, Phys. Lett. B **103** (1981) 207, Phys. Lett. B **103** (1981) 211.

[6] T. L. Curtright and C. B. Thorn, Phys. Rev. Lett. **48** (1982) 1309 [Erratum-ibid. **48** (1982) 1768].

[7] J. L. Gervais and A. Neveu, Nucl. Phys. B **209** (1982) 125.

[8] C. B. Thorn, Phys. Lett. B **242** (1990) 364.

[9] C. R. Preitschopf and C. B. Thorn, Nucl. Phys. B **349** (1991) 132.

[10] A. M. Polyakov, Int. J. Mod. Phys. A **14** (1999) 645 [arXiv:hep-th/9809057].

[11] S. Mandelstam, private communication, April 1971.

[12] F. Gliozzi, J. Scherk and D. I. Olive, Phys. Lett. B **65**, 282 (1976); Nucl. Phys. B **122** (1977) 253.

[13] A. Neveu and J. H. Schwarz, Nucl. Phys. B **31** (1971) 86.

[14] A. Neveu, J. H. Schwarz and C. B. Thorn, Phys. Lett. B **35** (1971) 529.

[15] C. B. Thorn, "Subcritical String and Large N QCD," arXiv:0808.0458 [hep-th].

[16] A. Neveu and J. Scherk, Nucl. Phys. B **36** (1972) 155.

[17] J. Dai, R. G. Leigh and J. Polchinski, Mod. Phys. Lett. A **4** (1989) 2073.

[18] R. C. Brower and C. B. Thorn, Nucl. Phys. B **31** (1971) 163.

[19] R. C. Brower, Phys. Rev. D **6** (1972) 1655.

[20] P. Goddard and C. B. Thorn, Phys. Lett. B **40** (1972) 235.

[21] I. B. Frenkel and V. G. Kac, Invent. Math. **62** (1980) 23; G. Segal, Commun. Math. Phys. **80** (1981) 301; P. Goddard and D. I. Olive,

[22] P. Goddard and R. E. Waltz, Nucl. Phys. B **34** (1971) 99.

[23] A. Chodos and C. B. Thorn, Nucl. Phys. B **72** (1974) 509.

[24] C. B. Thorn, Phys. Rept. **175** (1989) 1.

[25] P. Ramond, Phys. Rev. D **3** (1971) 2415; A. Neveu and J. H. Schwarz, Phys. Rev. D **4** (1971) 1109; C. B. Thorn, Phys. Rev. D **4** (1971) 1112.

[26] C. B. Thorn, unpublished comments, Santa Fe Institute workshop, November 8-10, 1985; N. Seiberg and E. Witten, Nucl. Phys. B **276** (1986) 272. L. J. Dixon and J. A. Harvey, Nucl. Phys. B **274** (1986) 93.

[27] I. R. Klebanov and A. A. Tseytlin, Nucl. Phys. B **546** (1999) 155 [arXiv:hep-th/9811035].

[28] P. Goddard, J. Goldstone, C. Rebbi and C. B. Thorn, Nucl. Phys. B **56** (1973) 109.

[29] S. Mandelstam, Nucl. Phys. B **64** (1973) 205; Nucl. Phys. B **69** (1974) 77.

[30] K. Bardakci and C. B. Thorn, Nucl. Phys. B **626** (2002) 287 [arXiv:hep-th/0110301]; C. B. Thorn, Nucl. Phys. B **637** (2002) 272 [Erratum-ibid. B **648** (2003) 457] [arXiv:hep-

th/0203167]; S. Gudmundsson, C. B. Thorn and T. A. Tran, Nucl. Phys. B **649** (2003) 3 [arXiv:hep-th/0209102].

[31] D. Chakrabarti, J. Qiu and C. B. Thorn, Phys. Rev. D **72** (2005) 065022 [arXiv:hep-th/0507280]; D. Chakrabarti, J. Qiu and C. B. Thorn, Phys. Rev. D **74** (2006) 045018 [Erratum-ibid. D **76** (2007) 089901] [arXiv:hep-th/0602026].

[32] P. Goddard, Nuovo Cim. A **4** (1971) 349; A. Neveu and J. Scherk, Nucl. Phys. B **36** (1972) 317.