Near-Integrability and Soft Scattering in QCD

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The first of the papers below discusses (3+1)-dimensional collisions. All the rest concern 2+1 dimensions.

Near-Integrability and Confinement for High-Energy Hadron-Hadron Collisions
Phys.Rev.**D77** (2008) 056004, arXiv:0801.0389

Composite Strings in (2+1)-Dimensional Anisotropic Weakly-Coupled Yang-Mills Theory
Phys.Rev.**D77** (2008) 025035, arXiv:0710.3733

Glueball Masses in (2+1)-Dimensional Anisotropic Weakly-Coupled Yang-Mills Theory
Phys.Rev.**D75** (2007) 101702, arXiv:0704.0940

String Tensions and Representations in Anisotropic 2+1-Dimensional Weakly-Coupled Yang-Mills Theory
Phys.Rev.D75 (2007) 025001, hep-th/0608067

Integrable Models and Confinement in (2+1)-Dimensional Weakly-Coupled Yang-Mills Theory
Phys.Rev.**D74** (2006) 085001, hep-th/0607013

Lattice (QCD)(2+1)

Phys.Rev.**D71** (2005) 054503, hep-lat/0501026

Some new work with J. Xiao on anisotropic renormalization should appear soon.

General Features of Hadron Collisions

At very high energies, the total cross section is dominated by diffraction (small x, small t).

The form of the total cross section is

$$\sigma^{\mathrm{TOT}} \sim s^{\epsilon}$$

where $\epsilon = 0.08$ (critical exponent).

Longitudinally rescaled QCD

Verlinde + Verlinde ('93),
McLerran and Venugopalan ('94)

$$x^L = (x^0, x^3) \ , \quad x^\perp = (x^1, x^2)$$
$$x^L \to \lambda x^L, \quad x^\perp \to x^\perp$$

The center-of-mass energy squared changes as $s \to \lambda^{-2} s$. Since we wish to consider high energies, we take $\lambda \ll 1$.

Rescaled action:

$$S = \frac{1}{2g_0^2} \int d^4x \, \mathrm{Tr}\Big(\sum_{j=1}^{2} F_{0j}^2 - \sum_{j=1}^{2} F_{j3}^2$$

$$+ \lambda^{-2} F_{03}^2 - \lambda^2 F_{12}^2 \Big),$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - \mathrm{i}[A_\mu, A_\nu].$$

Hamiltonian:

$$H = \int d^3x \left[\frac{g_0^2}{2}\mathcal{E}_\perp^2 + \frac{1}{2g_0^2}\mathcal{B}_\perp^2 \right.$$

$$\left. +\lambda^2 (\frac{g_0^2}{2}\mathcal{E}_3^2 + \frac{1}{2g_0^2}\mathcal{B}_3^2) \right] \ ,$$

$$(\partial_\perp \cdot \mathcal{E}_\perp + \partial_3 \mathcal{E}_3 - \rho) \, \Psi_{\rm Physical} = 0 \ ,$$

Gauge-invariant UV cut-off and $\lambda \ll 1$ $\Longrightarrow$ mass gap and confinement.

Futhermore $\lambda = 0 \Longrightarrow$ integrability.

Lattice Hamiltonian:

Gauge field: $U_j(x) \in \mathrm{SU}(N)$

El. field: $[l_j(x)_b, U_k(y)] = -\delta_{xy}\delta_{jk}t_b U_j(x)$

Hamiltonian: $H = H_0 + H' + H''$

$$H_0 = \frac{1}{a}\sum_x \left[\frac{g_0^2}{2} l_\perp(x)^2 - \frac{1}{4g_0^2}\,\mathrm{Tr}\,U_\perp^\square(x)\right],$$

$$H' = \frac{(g_0')^2}{2a}\sum_x l_3(x)^2, \quad g_0' = \lambda g_0,$$

$$H'' = -\frac{1}{4(g_0'')^2 a}\sum_x \mathrm{Tr}\,U_3^\square(x),\ g_0'' = \lambda^{-1} g_0,$$

where

$$U_i^\square(x) = \epsilon^{ijk} U_j(x) U_k(x+\hat{j}a) U_j(x+\hat{k}a)^\dagger U_k(x)^\dagger.$$

H_0 is a set of 1+1-dimensional $\mathrm{SU}(N)$ principal-chiral sigma models,

$$\mathcal{L} = \frac{1}{2g_0^2}\int d^2x \, \mathrm{Tr}\, \partial^\mu U^\dagger \partial_\mu U, \quad \mu = 0, 3,$$

hence the $\lambda \longrightarrow 0$ limit of the gauge theory, is completely integrable.

We can treat $H' + H''$ as an interaction Hamiltonian, provided

$$\lambda^2 \ll g_0^{-3} \exp - \frac{4\pi}{g_0^2 N}.$$

1+1-dimensional PC Sigma Model

Mass Spectrum:

$$m_r = m_1 \frac{\sin(\pi r/N)}{\sin(\pi/N)},\ r = 1, \ldots, N-1$$

"Elementary" soliton-like particles have $r = 1$ and are color dipoles (like a quark-antiquark pair). Other ($r > 1$) particles are bound states. The elementary antiparticle has $r = N - 1$.

(r=1) by (r=1) S-matrix:

$$\mathfrak{S}_{11}(\theta) = \frac{\sin(\theta/2 - \pi \mathrm{i}/N)}{\sin(\theta/2 + \pi \mathrm{i}/N)}\ S_{\mathrm{CGN}}(\theta) \otimes S_{\mathrm{CGN}}(\theta)$$

where

$$S_{\mathrm{CGN}}(\theta) = \frac{\Gamma(\mathrm{i}\theta/2\pi + 1)\Gamma(-\mathrm{i}\theta/2\pi - 1/N)}{\Gamma(\mathrm{i}\theta/2\pi + 1 - 1/N)\Gamma(-\mathrm{i}\theta/2\pi)}(\mathbb{1} - \frac{2\pi \mathrm{i}}{\theta} P).$$

Other S-matrix elements are found by fusion and crossing ($\theta \to \pi\mathrm{i} - \theta$).

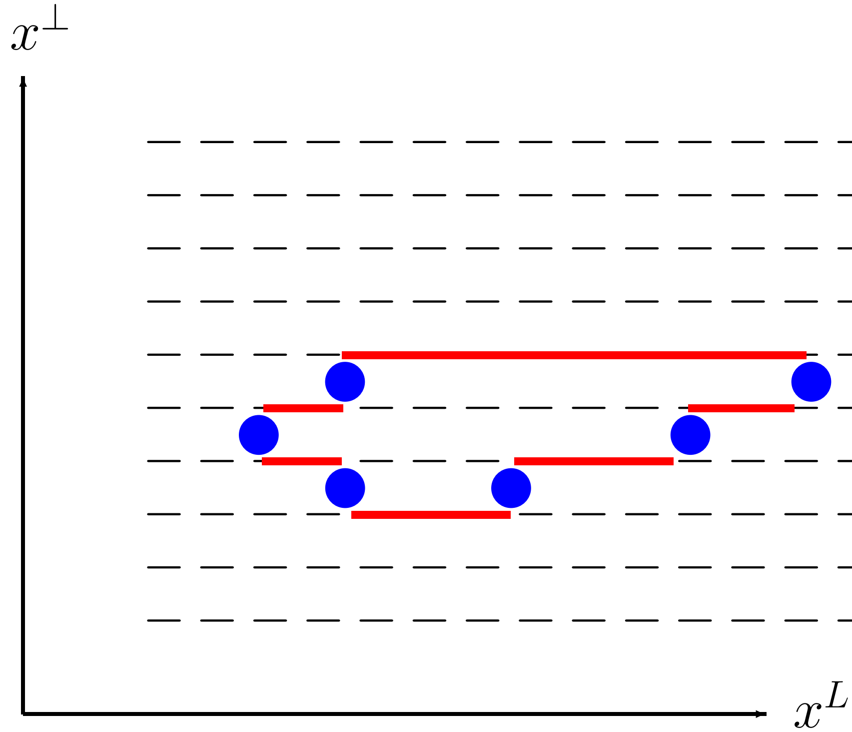
Gauss Law + Mass Gap $\Longrightarrow$ Confinement

One coupling g_0'', is strong, so we haven't proved confinement in (3+1)-dimensional QCD. In 2+1 dimensions, there is no g_0'', and confinement takes place at weak coupling.

Diffraction:

If $\lambda = 0$, particles move only longitudinally, not transversely, and scattering is only in the forward direction. As λ is increased, the diffraction peak broadens.

Glueball (shown only for SU(2))



The blue bullets are soliton-like excitations. The red lines are longitudinal electric flux.

Meson and baryon states are similar, except that flux can terminate on a quark.

Summary of Results for d=2+1 and $N = 2$

Longitudinal string tension:

$$\sigma_{\rm L} = \frac{3(g_0')^4}{8K},$$

$$K = \frac{(g_0')^2 a^2}{4} + \frac{1}{3m^2\pi^2}\left(\frac{g_0'}{g_0}\right)^4 \exp\left[-2\int_0^\infty \frac{d\xi}{\xi} e^{-\xi} \tanh^2\frac{\xi}{2}\right].$$

Transverse string tension:

$$\sigma_\perp = \frac{m}{a} - \frac{2\sqrt{3}}{\pi}\frac{g_0'}{g_0^2 a^2}.$$

An exact form factor of the sigma model was used to find these string tension corrections.

Summary of Results for d=2+1 and $N = 2$ (CONTINUED)

Low-lying mass spectrum:

$$M_n = 2m_1 + \left[\epsilon_n^{1/3} - \frac{3(3-2\ln 2)\sigma_\perp}{4\pi m}\epsilon_n^{-1/3}\right]^2,$$

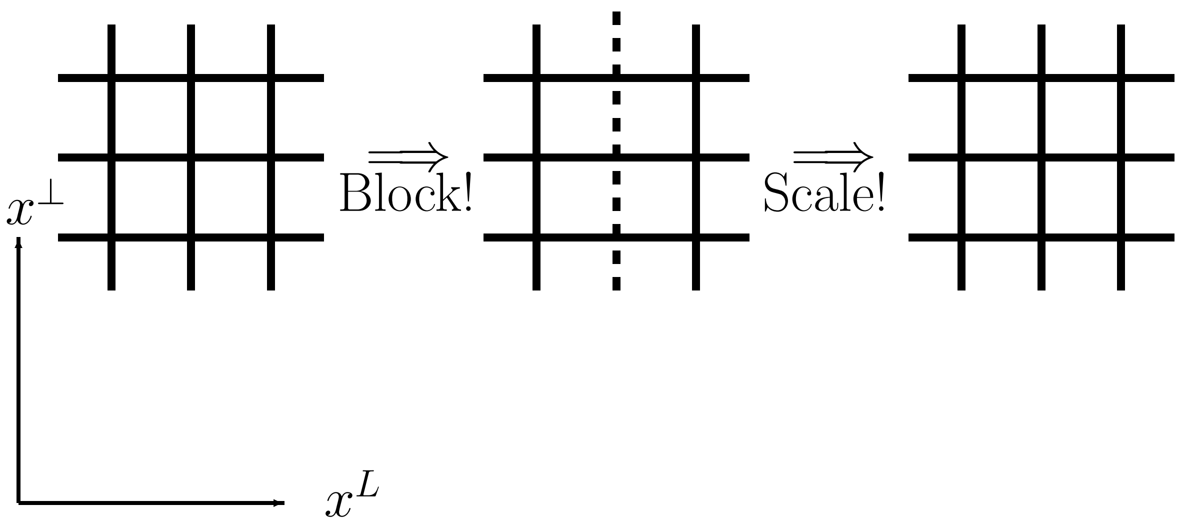
$$\epsilon_n = \frac{3\pi\sigma_\perp(n+\frac{1}{2})}{4m^{1/2}} + \left\{\left[\frac{3\pi\sigma_\perp}{4m^{1/2}(n+\frac{1}{2})}\right]^2 + \frac{1}{8}\left[\frac{3(3-2\ln 2)\sigma_\perp}{2\pi m}\right]^3\right\}^{1/2}.$$

k-string tensions:

Casimir law for longitudinal string tensions.
Sine law for transverse string tensions.

The rescaling $x^L \to \lambda x^L$, is classical. How can we do a quantum rescaling?

Consider $\lambda = 1/2$. We integrate out degrees of freedom to get an effectively-doubled longitudinal lattice spacing. Then we rescale.



In practice, it is easier to perform the first step in perturbation theory. This has been done for SU(N) gauge theory (with J. Xiao). We have not included quark fields yet.

Split A_μ into slow components $\tilde{A}_\mu$ and fast components a_μ: $A_\mu = \tilde{A}_\mu + a_\mu$.

$$A_\mu(x) = \int_{p^2 < \Lambda^2} A_\mu(p) e^{-\mathrm{i}p\cdot x} \frac{d^4p}{(2\pi)^4},$$

$$A_\mu(x) = \int_{bp_L^2 + p_\perp^2 < \tilde{\Lambda}^2} A_\mu(p) e^{-\mathrm{i}p\cdot x} \frac{d^4p}{(2\pi)^4},$$

$$\tilde{\Lambda} \leq \Lambda,\ b > 1.$$

The parameter b is the anisotropy. We set $\lambda^2 b = 1$. After integrating out the fast degrees of freedom, rescale $x^L \to \lambda x^L$.

Wilsonian R.G.

$$Z_\Lambda = \int_{p^2<\Lambda^2} [dA(p)] e^{-S-S_{\text{counter.},\Lambda}}$$

$$= e^{-f} \int_{bp_L^2+p_\perp^2<\tilde{\Lambda}^2} [dA(p)] e^{-\tilde{S}-\tilde{S}_{\text{counter.},\tilde{\Lambda},b}}$$

The only real subtlety is separating the (maximally non-gauge-invariant) counterterm action from the part of the action depending only on $F_{\mu\nu}$, or $\tilde{F}_{\mu\nu}$.

Find $\tilde{S}$, then rescale. This gives the final form of $\tilde{S}$ (abuse of notation).

The effective action (after renormalization and rescaling) is:

$$\tilde{S} = \int d^4x \sum_{\mu\nu} \frac{1}{4\tilde{g}_{\mu\nu}} \mathrm{Tr} \tilde{F}_{\mu\nu} \tilde{F}^{\mu\nu}$$

Coupling in the Effective Action

$$\tilde{g}_{L\perp} = \tilde{g}_{01} = \tilde{g}_{02} = \tilde{g}_{13} = \tilde{g}_{23},$$

$$\frac{1}{\tilde{g}_{L\perp}^2} = \frac{1}{g^2} - \frac{11C_N}{96\pi^2} \ln \frac{\Lambda}{\tilde{\Lambda}} - \frac{13C_N}{384\pi^2} \ln b,$$

$$\frac{1}{\tilde{g}_{03}^2} = \frac{1}{\tilde{g}_{L\perp}^2} + \frac{C_N}{512\pi^2} \ln b,$$

$$\frac{1}{\tilde{g}_{12}^2} = \frac{1}{\tilde{g}_{L\perp}^2} + \frac{13C_N}{1536\pi^2} \ln b.$$

This yields anomalous rescaling of the couplings.

Quantum Effective Action at Large s

$$\tilde{S} =$$

$$\frac{1}{2\tilde{g}_{L\perp}^2}\lambda^{\frac{13C_N\tilde{g}_{L\perp}^2}{192\pi^2}} \int d^4x \mathrm{Tr}\left[\sum_{j=1}^{2}(F_{0j}^2 - F_{j3}^2) + \lambda^{-2-\frac{C_N\tilde{g}_{L\perp}^2}{256\pi^2}}\tilde{F}_{03}^2 - \lambda^{2-\frac{13C_N\tilde{g}_{L\perp}^2}{768\pi^2}}\tilde{F}_{12}^2\right]$$

So compared with their classical values, g_0' is even slightly smaller, and g_0'' is also a bit smaller.

Comments

The calculation of anomalous dimensions could also have been done by deriving the Callan-Symanzik eq. of the operator generating longitudinal rescaling (in dimensional regularization, for example).

To get the effective action, we have to integrate out a lot of degrees of freedom. So we haven't proved our effective action is correct, any more than we can prove strongly-coupled lattice gauge theory or AdS/QCD is correct. The best we can hope for is gaining some qualitative understanding. In 2+1 dimensions, the situation is much better.

Problems Under Investigation

Exact SU(3) sigma-model form factors can be found. This would be a start in comparing with BFKL theory.

To find the forward elastic scattering amplitude, we need the distribution of soliton-like particles in the transverse plane and in rapidity space. In other words, we need to work out hadron states. How far can we get without solving this complicated problem?

Can we establish confinement, once and for all, at weak (dimensionless) coupling in 2+1 dimensions (assuming the validity of the exact results for sigma models).