Matrix models for the black hole information paradox

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Joint work with N. Iizuka and J. Polchinski

Black hole information paradox

- Hawking's paradox for evaporating black holes
- Maldacena's paradox for eternal black holes

Matrix models for black holes

- Exponential decay of a two-point function (review)
- Perturbative 1/N corrections do not restore information

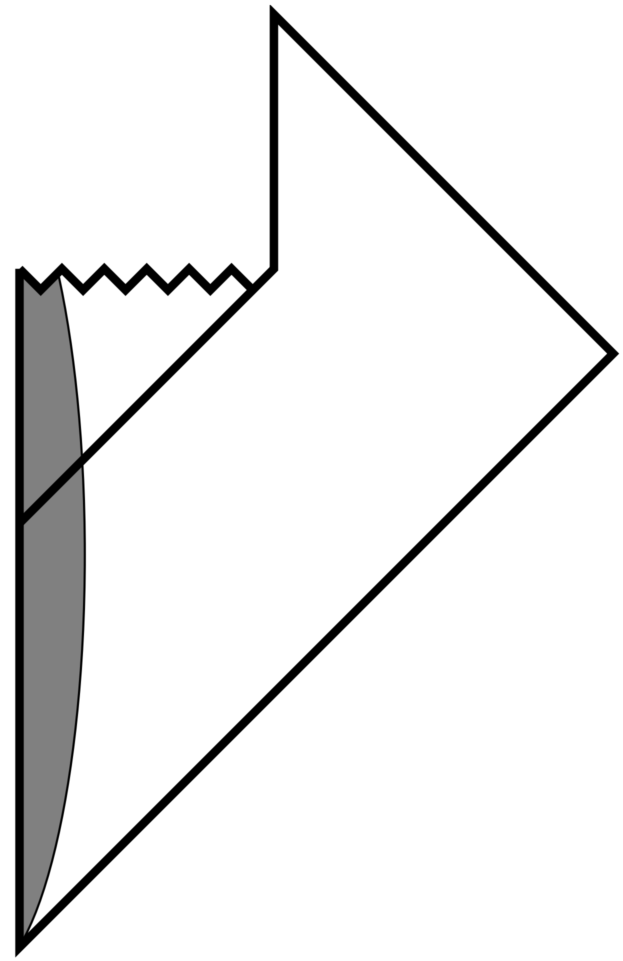
Hawking argued that information is not conserved (information paradox)

Collapsing matter produces a black hole.

A black hole in Minkowski space evaporates by emitting radiation.

Semiclassically, radiation is thermal, and there is no phase correlation.

Information seems to be lost.

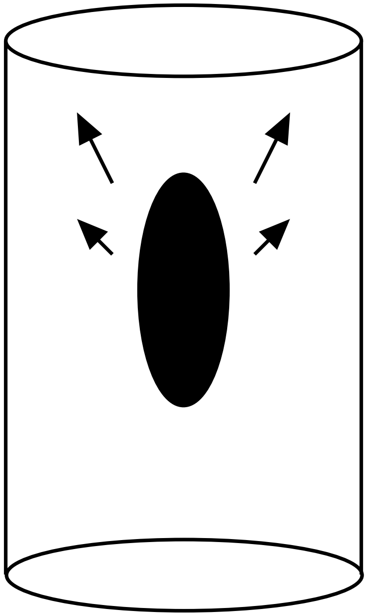


AdS/CFT taught us that information is preserved

A **small black hole** in AdS evaporates, and the process is described, in principle, by a unitary evolution in gauge theory.

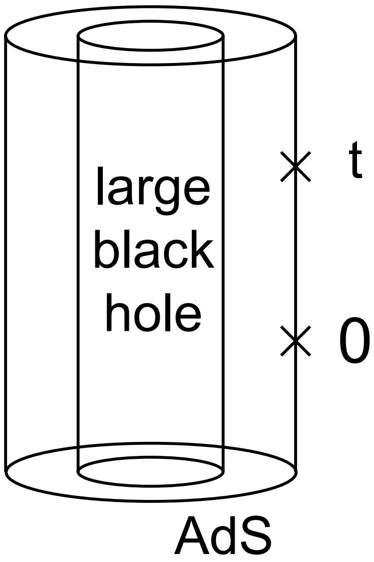
Information must be preserved.

There have to be phase correlations. We want to understand **where Hawking's argument breaks down**.



Mardacena's ``paradox'' is a cleaner set-up

A **large black hole** in AdS does not evaporate, so there is no information paradox of Hawking. A related ``paradox'': A two-point correlation function <O(t)O(0)> shows **exponential fall-off** $e^{-\Gamma t}$ in AdS. Information lost at large N.



In gauge theory, there must be **recurrences** (discrete spectrum). Information restored at finite N.

It's not a true paradox:

Exponential decay at large N, **recurrences** at finite N.

Questions in gravity are difficult

Questions:

Which correction in gravity restores recurrences?
(Perturbative loop corrections or non-perturbative?)

Is it the secondary saddle points in Euclidean gravity?

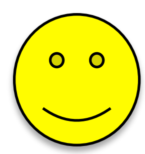
~~$G_{\mu\nu}$~~

Questions in gauge theory are easier

Questions:

Which correction in gauge theory restores recurrences?
(Perturbative 1/N corrections or non-perturbative in N?)

Is it the secondary saddle points in Polyakov loop integral?
(Aharony et al.)

A_μ 

Ask the questions in toy matrix models

Realistic theories like N=4 Yang-Mills are still too difficult.

→ Two toy models
(Matrix quantum mechanics)

Goals:

1. Exponential decay (Iizuka and Polchinski)
2. 1/N corrections (Iizuka, TO and Polchinski)

Study a cubic model to demonstrate exponential decay (Iizuka and Polchinski)

The simplest possible model:

One adjoint field $X \propto A - A^\dagger$

One fundamental field $\phi \propto a - a^\dagger$

The Hamiltonian is

$$H = mA^\dagger_{ij}A_{ji} + Ma^\dagger_i a_i + g a^\dagger_i (A + A^\dagger)_{ij} a_j$$

Consider the two-point function at finite T

$$e^{iM(t-t')} \left\langle \mathrm{T}\, a_i(t) a^\dagger_j(t') \right\rangle_T \equiv \delta_{ij} G(T, t-t')$$

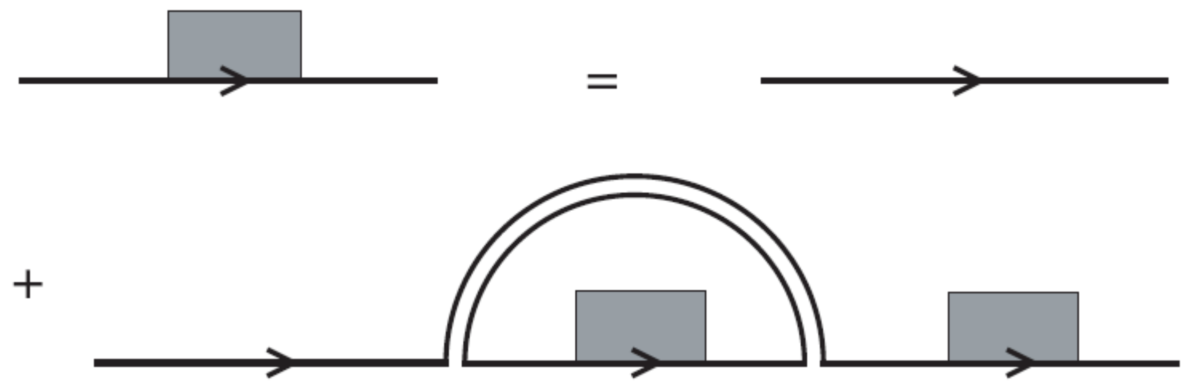
Recurrences at finite N are automatic in matrix quantum mechanics

Demonstration of exponential decay in a two-point function at large N is non-trivial.

They worked in the planar approximation.

The key tool is the planar Schwinger-Dyson equation

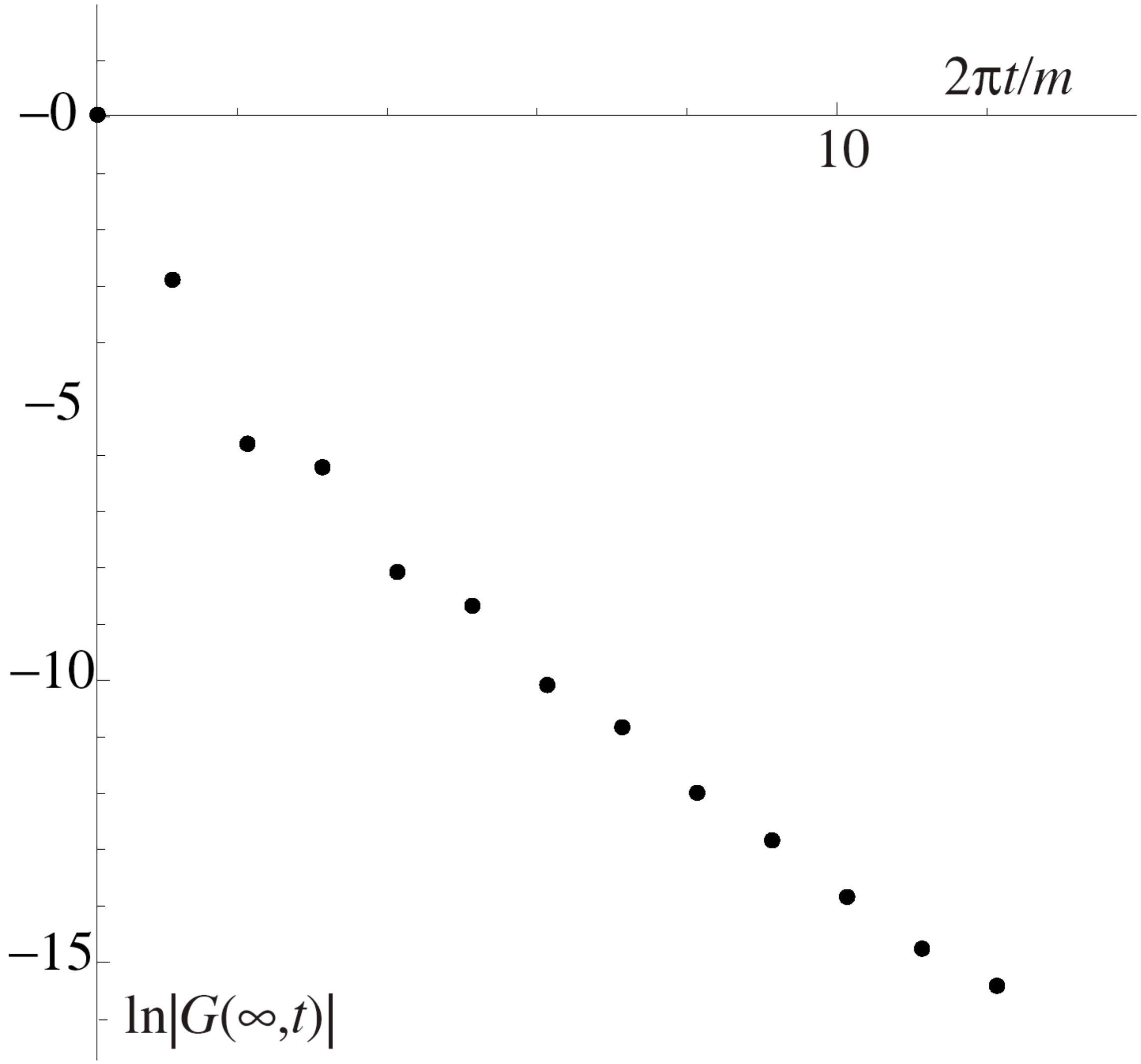
Graphically, the SD equation is given as



We can prove it within the radius of convergence (going to zero at late times), then analytically continue.

The planar Schwinger-Dyson equation is valid at any time and an value of the ’t Hooft coupling g^2N.

They demonstrated exponential decay at high temperature



Mini-summary for the cubic model
(Iizuka and Polchinski)

Demonstrated exponential decay at high temperature.

Numerical results.

Analytic solutions desired.

Study a charge-charge model to compute perturbative 1/N corrections (Iizuka, TO, Polchinski)

The Hamiltonian is

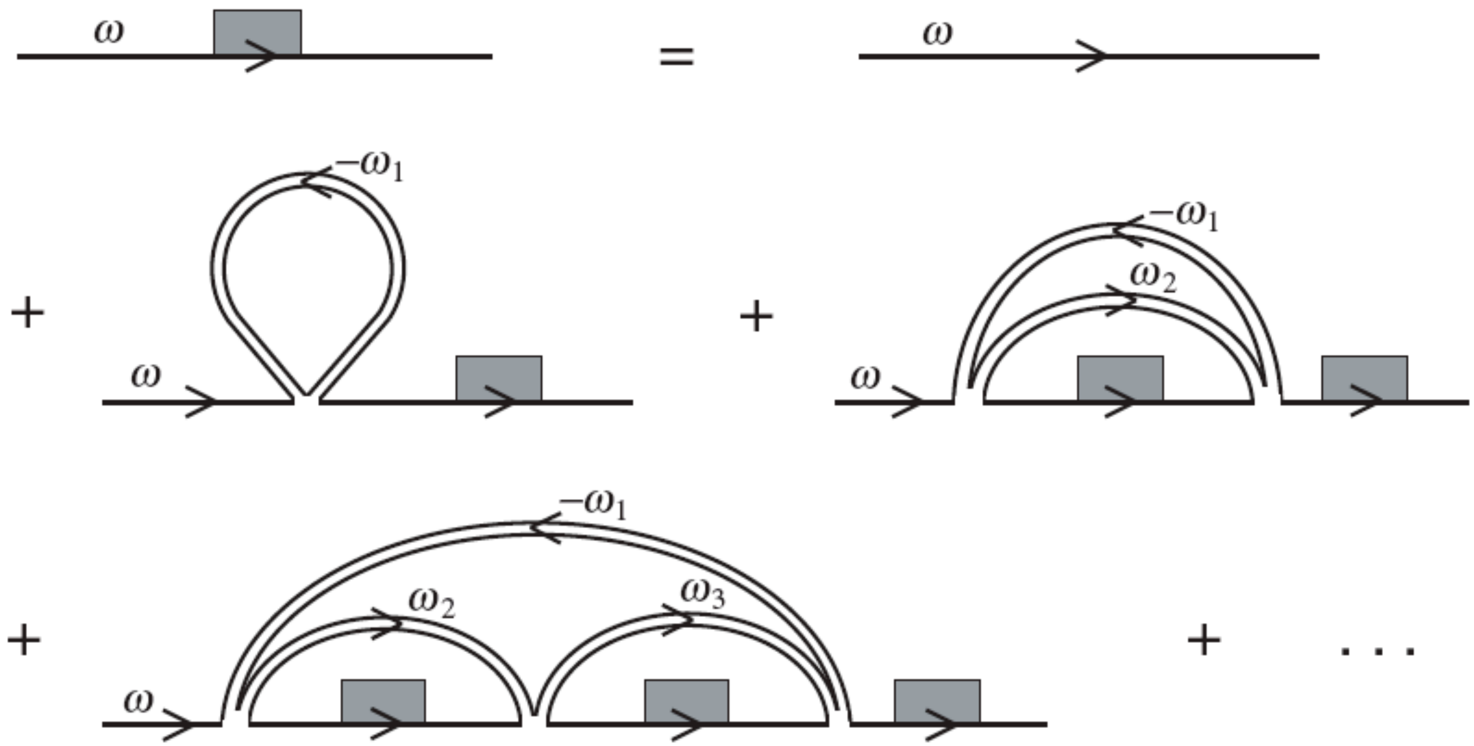
$$H = mA^{\dagger}_{ij}A_{ji} + Ma^{\dagger}_{i}a_{i} + H_{\text{int}}$$

$$H_{\text{int}} = -hq_{li}Q_{il} \ , \quad Q_{il} = A^{\dagger}_{ij}A_{jl}$$
$$q_{li} = -a^{\dagger}_{i}a_{l}$$

The model has a large amount of symmetries, and is not generic.

We used Feynman diagrams and the SD equations to compute corrections

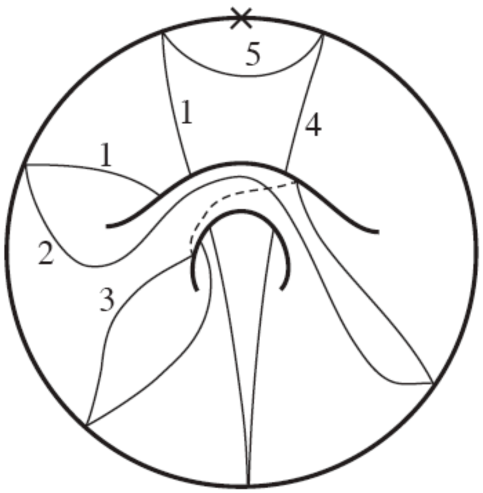
Genus zero

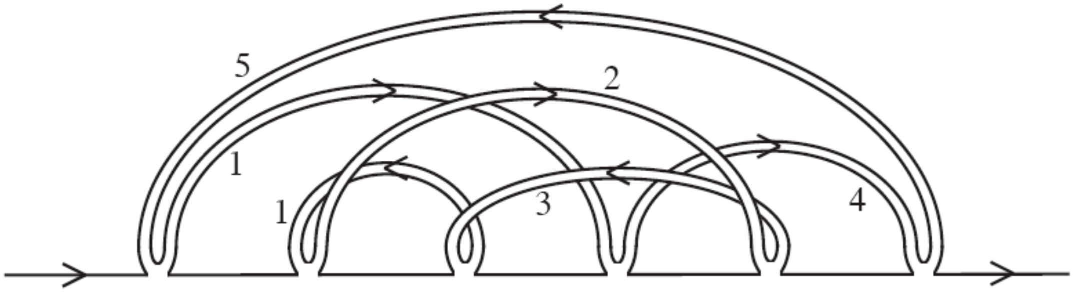


We used Feynman diagrams and the SD equations to compute corrections

The diagrams for genus one can be enumerated.

The genus zero and one SD equations can be solved.





Continuous spectrum remains after a 1/N correction is included

Genus zero: a branch cut

$$\tilde{G}^{(0)}(T,\omega) = \frac{i(1-y)}{2\omega\lambda}\left(\lambda + \omega - \sqrt{(\omega-\omega_+)(\omega-\omega_-)}\right)$$

Genus one: still a branch cut = continuous spectrum

$$\tilde{G}^{(1)}(T,\omega) = \frac{iy^2 x_0^3(1-x_0)^4(1-x_0[1-y])}{(1-2x_0+x_0^2[1-y])^4(\omega[1-x_0]^2-\lambda_y y)}$$

$$x_0 = -i\lambda_y \tilde{G}^{(0)}(\omega) \qquad \lambda_y = \frac{\lambda}{1-y} = \frac{hN}{1-y}$$

Mini-summary for the charge-charge model:

The genus zero two-point function has a continuous spectrum and decays by a power-law.

The genus one function also has a continuous spectrum, so does not restore recurrences. (It actually grows by a power-law. We think it's an artifact.)

We also developed other techniques that are suggestive of the gravity picture.

What restores information?

Maldacena and Hawking conjectured that the sum over geometries (saddle points) restores information.

A known saddle point is the thermal AdS, which has the same boundary as the AdS black hole (Hawking & Page).

The thermal AdS is expected to be realized as a saddle in the gauge field integral = Polyakov loop. (Aharony et al.)

What restores information?

We didn't include the gauge field, so the second saddle is not the reason for information restoration.

There are other works arguing against the conjecture.
(Barbon, Rabinovic, Birmingham, Sachs, Solodukhin, Kleban, Porrati, Rabadan, …)

Summary and conclusions

AdS black holes provide cleaner set-ups to study the black hole information paradox.

Iizuka and Polchinski demonstrated exponential decay in a large N matrix model.

Perturbative 1/N corrections do not resolve information loss, which requires non-perturbative effects.

Our models are examples where recurrences are restored without extra saddle points.