

Quantum Wires and Simplexes in an Integrable System

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SYNOPSIS

- ▶ The fundamental vertex of delta interaction may be interpreted as a single qubit quantum gate that provides a unitary connection among four amplitudes arranged on two “wires” in and two wires out.
- ▶ The integrability of delta interaction allows these gates to be chained in a space-time triangle, a simplex.
- ▶ The unitary connection is re-interpreted (by crossing symmetry) into a gate that connects the sheets of a multiply connected Riemann surface.
- ▶ This reformulation allows accessible computation in the limit of a large number of degrees of freedom, i.e. a reduction in complexity from $N!$ to N .

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DELTA INTERACTION

Delta interaction is an integrable quantum system composed of equivalent point particles in $1(\text{space})+1(\text{time})$ dimensions.

DYNAMICS

1. Particles travel freely (without acceleration) between elastic collisions that occur when particle coordinates are equal.
2. The particles are distinguishable and their identity is preserved in a collision.
3. A single particle “travels” on a straight world line or “quantum wire”, a line of constant slope in space-time.
4. Two particles travel on two straight world lines that intersect at a single point in space-time, a vertex of the quantum wires.
5. Three particles travel on three straight world lines whose intersections lie at the vertices of a triangle in space-time, a simplex.

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TWO PARTICLE PROBLEM 1

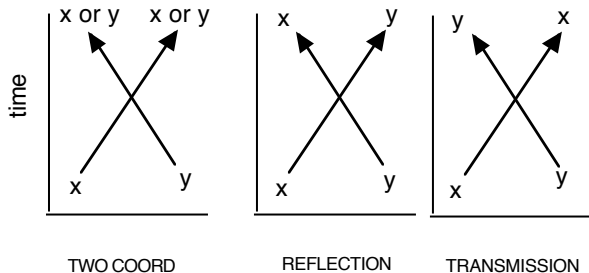


Figure: Two particle diagram

TWO PARTICLE PROBLEM 2

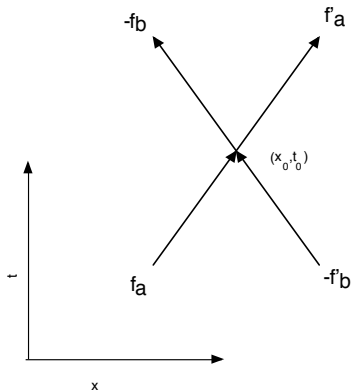


Figure: A delta interaction encounter in space-time

$$\Psi(x, y) = \begin{cases} f_a e^{ik_a x} e^{ik_b y} - f_b e^{ik_b x} e^{ik_a y} & x < y \\ f'_a e^{ik_a x} e^{ik_b y} - f'_b e^{ik_b x} e^{ik_a y} & x > y \end{cases}$$

TWO PARTICLE PROBLEM 3

$$\begin{pmatrix} f'_a \\ -f'_b \end{pmatrix} = \begin{pmatrix} T_{ab} & -R_{ab} \\ -R_{ab} & T_{ab} \end{pmatrix} \begin{pmatrix} f_a \\ -f_b \end{pmatrix},$$

The events "transmission" and "reflection" are complementary

$$TT^* + RR^* = 1.$$

The events "transmission" and "reflection" are mutually exclusive

$$TR^* + RT^* = 0.,$$

and

$$\frac{T}{R} = -\frac{T^*}{R^*}$$

This "transfer matrix" is unitary and symmetric (because the particles are equivalent)

THE THREE PARTICLE PROBLEM 1

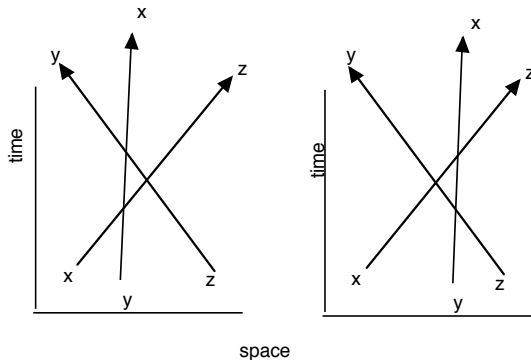


Figure: Three particle simplexes

Amplitude for simplex on left=Amplitude for simplex on right

$$T_{ab}R_{bc}R_{ac} + R_{ab}R_{bc}T_{ac} = R_{ab}T_{bc}R_{ac}$$

THE THREE PARTICLE PROBLEM 2

$$\frac{T_{ab}}{R_{ab}} + \frac{T_{bc}}{R_{bc}} = \frac{T_{ac}}{R_{ac}}.$$

Satisfied if

$$\frac{T_{ab}}{R_{ab}} = \frac{i(k_a - k_b)}{g} = i\rho_{ab}$$

$$T = \frac{i\rho}{i\rho - 1} R = \frac{1}{i\rho + 1}$$

THE TWO PARTICLE PROBLEM REDUX 1

The two particle equations are two relations among four amplitudes:

$$\begin{pmatrix} f'_a \\ -f'_b \end{pmatrix} = \begin{pmatrix} T_{ab} & -R_{ab} \\ -R_{ab} & T_{ab} \end{pmatrix} \begin{pmatrix} f_a \\ -f_b \end{pmatrix}$$

The relations contain the same information if this matrix equation is multiplied by any 2×2 matrix, or if the matrix is conjugated with any unitary transformation.

THE TWO PARTICLE PROBLEM REDUX 2

The same information is also contained if we rewrite the matrix using crossing symmetry

$$\begin{pmatrix} f'_a \\ f_a \end{pmatrix} = \begin{pmatrix} 1 - \frac{i(k_b - k_a)}{g} & \frac{i(k_b - k_a)}{g} \\ -\frac{i(k_b - k_a)}{g} & 1 + \frac{i(k_b - k_a)}{g} \end{pmatrix} \begin{pmatrix} f'_b \\ f_b \end{pmatrix}.$$

The 2×2 "transfer matrix" factors into a pair of commuting matrices, leading to a separation of a, b .

$$\tau(k_a - c_{xy}) \begin{pmatrix} f'(k_a) \\ f(k_a) \end{pmatrix} = \tau(k_b - c_{xy}) \begin{pmatrix} f'(k_b) \\ f(k_b) \end{pmatrix} = \begin{pmatrix} s_{xy} \\ 1 \end{pmatrix}$$

and c_{xy}, s_{xy} are separation constants associated with coordinates x, y .

THE TWO PARTICLE PROBLEM REDUX 3

$$\tau(k - c) = \begin{pmatrix} 1 - \frac{i(k-c)}{g} & \frac{i(k-c)}{g} \\ -\frac{i(k-c)}{g} & 1 + \frac{i(k-c)}{g} \end{pmatrix}.$$

Thus,

$$\frac{f'(k)}{f(k)} = \frac{s + \frac{i(k-c)}{g}(s-1)}{1 + \frac{i(k-c)}{g}(s-1)} = \nu(k).$$

The separation constant $s^2 = 1$, because ν is a phase. If $s=1$,
 $\nu=1$ if $s = -1$

$$\frac{f'(k)}{f(k)} = \frac{\frac{2i(k-c)}{g} + 1}{\frac{2i(k-c)}{g} - 1} = \nu(k).$$

CONCLUSION

1. No matter how many degrees of freedom the problem is reducible to one particle simplexes.

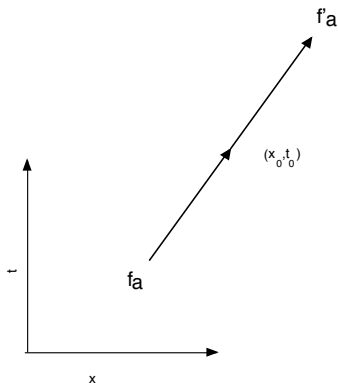


Figure: One particle simplex

2. This is a reduction from $N!$ regions of a simply connected region to N sheets of a multiply connected Riemann surface.