Effect of NUT parameter on the analytic extension of the Cauchy horizon that develop in colliding wave spacetimes

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1 Outline

→ Introduction
→ A new extension of the Chandrasekhar-Xanthopoulos colliding wave solution
→ Analytic extension of the spacetime across the Cauchy horizon
→ Concluding remarks

2 Introduction.

→ Black holes (BHs) and Gravitational waves (GWs) are the most important predictions of the Einstein's theory of general relativity.


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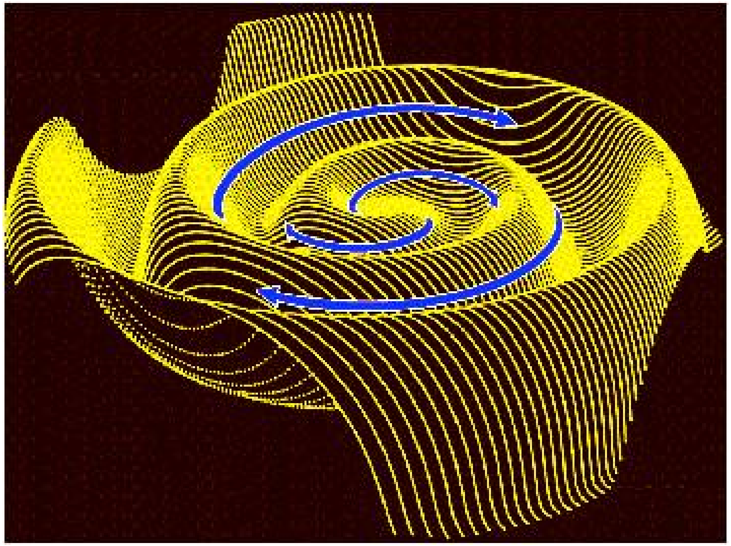
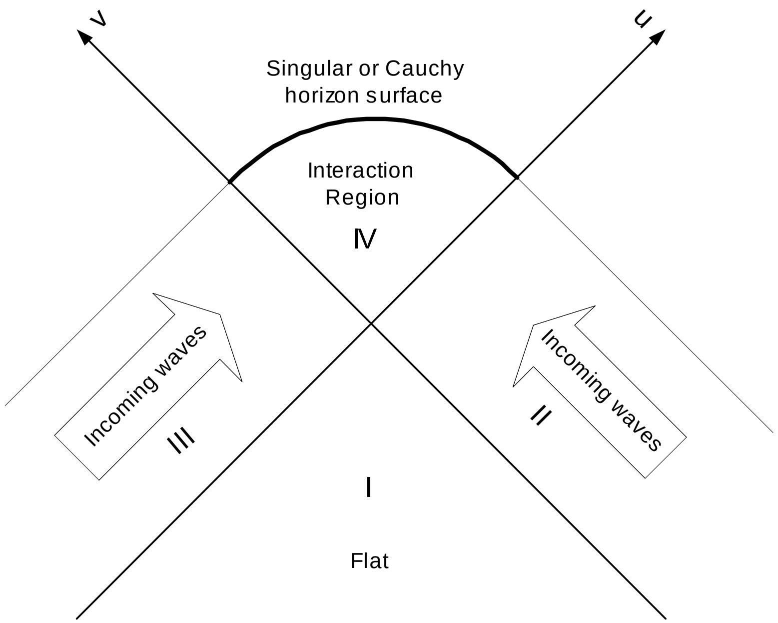


→ The common feature of the colliding gravitational waves (CGWs) is the occurence of space-like curvature singularities.
→ However, some exceptional solutions do not exhibit space-like curvature singularities, instead they develop horizons.

→ Chandrasekhar and Xanthopoulos (CX) (*Proc. Roy. Soc. London,* **A408**, 175, 1986, hereafter this will be referred as Paper I) have shown an interesting duality relation between BHs and CGWs.
→ Paper I, describes the collision of impulsive gravitational waves accompanied with shock gravitational waves.



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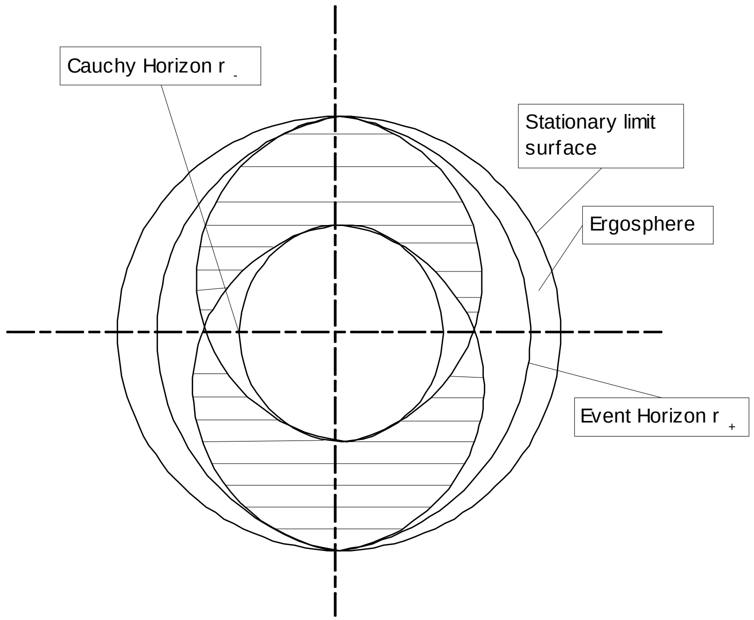
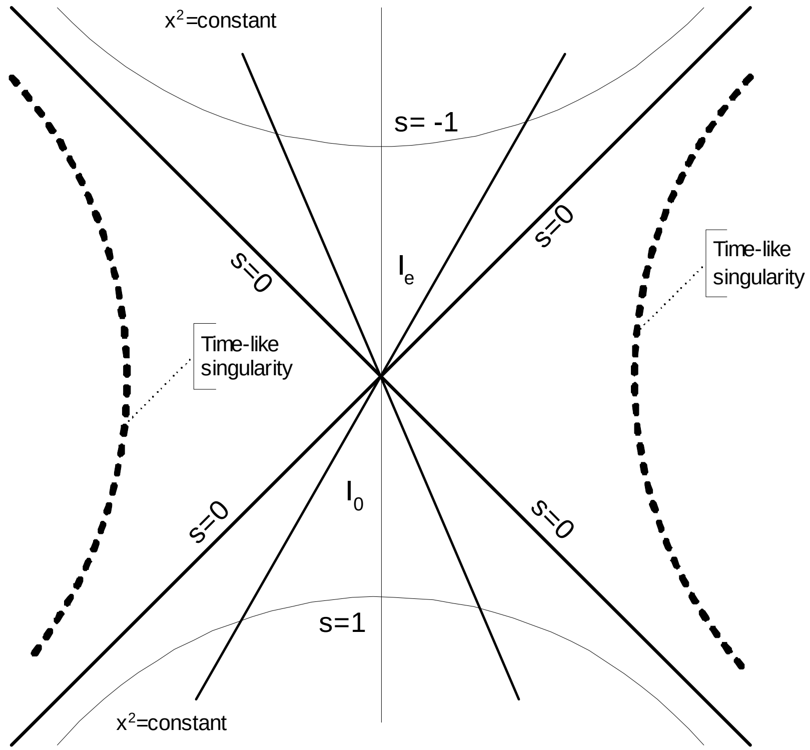


→ The remarkable aspect of Paper I is that, the two seemingly unrelated topics BHs and CGWs is shown to be locally isometric.
→ Einstein-Maxwell extension of Paper I is given by CX (*Proc. Roy. Soc. London,* **A414**, 1, 1987, hereafter this will be referred as Paper II)

→ Non-unique analytic extension beyond the horizon has revealed that two-dimensional timelike singularities along hyperbolic arcs are developed.



$\rightarrow$ The research in the BH area has extended by adding different kind of sources like,

Cosmological constant (Λ)
Acceleration parameter (α)
NUT(Newman-Unti-Tamburino)parameter (l)

$\rightarrow$ Our interest relies completely on the isometric equivalence of the extended BH solutions with CGW metrics that admits horizons.

Λ $\rightarrow$ correspond null-shells in the space of colliding waves $\rightarrow$ creates curvature singularity
α $\rightarrow$ isometric equivalence possible
l $\rightarrow$ admit horizon in the space of colliding waves

3 A New Extension of the CX Colliding Wave Solution.

→The metric describing the KNN black hole is given by ,

$$ds^2 = \frac{U^2}{\rho^2}(dt - Pd\phi)^2 - \frac{\sin^2\theta}{\rho^2}\left[\left(F + l^2\right)d\phi - adt\right]^2 - \frac{\rho^2}{U^2}dr^2 - \rho^2 d\theta^2 \tag{1}$$

where

$$F = r^2 + a^2, \quad U^2 = r^2 - 2mr + a^2 + Q^2 - l^2, \quad \rho^2 = r^2 + \lambda^2,$$

$$P = a\sin^2\theta - 2l\cos\theta, \quad \lambda = l + a\cos\theta.$$

→ The constants a, Q and l stand for the rotation, electric charge and NUT parameters, respectively.
→ The corresponding CGW metric can be determined by using the transformation

$$r = m + \sqrt{m^2 + l^2 - a^2 - Q^2}\tau, \quad \sigma = \cos\theta, \quad t = x, \quad \phi = y, \tag{2}$$

→ The CGW metric can be cast in the form (and after setting the mass parameter $m = 1$),

$$ds^2 = X\left(\frac{d\tau^2}{\Delta} - \frac{d\sigma^2}{\delta}\right) - X^{-1}\left(Rdx^2 + Edy^2 - 2Gdxdy\right). \tag{3}$$

→ Our notation and abbreviations in this metric are as follow

$$\begin{aligned} X &= (p+\tau)^2 + (l_0 + a_0\sigma)^2 = B - a_0 A, \\ R &= \Delta + a_0^2\delta, \qquad E = \Delta A^2 + \delta B^2, \qquad G = \Delta A + a_0\delta B, \\ A &= a_0\delta - 2l_0\sigma, \qquad B = (p+\tau)^2 + a_0^2 + l_0^2, \end{aligned} \tag{4}$$

where

$$\begin{aligned} \Delta &= 1-\tau^2, \qquad \delta = 1-\sigma^2, \\ \tau &= \sin\left(\widetilde{a}u + \widetilde{b}v\right), \qquad \sigma = \sin\left(\widetilde{a}u - \widetilde{b}v\right), \qquad \text{and} \qquad \widetilde{a}, \widetilde{b} = \textit{ constant}, \\ l_0 &= pl, \qquad a_0 = ap, \qquad q = Qp, \end{aligned} \tag{5}$$

in which $$p = \frac{1}{\sqrt{1+l^2-a^2-Q^2}} \quad \text{so that} \quad p^2 + l_0^2 - a_0^2 - q^2 = 1. \tag{6}$$

→ The NUT parameter l is constrained by $1 + l^2 > a^2 + Q^2$. To interpret the foregoing metric as CGW: $u \to u\theta(u)$ and $v \to v\theta(v)$. θ →Heaviside step fnc.

The non zero Ricci components due to the em field in the Newman - Penrose formalism are given, after tedious calculation by

$$
\begin{aligned}
\Phi_{22} &= \theta(u) q^2 X^{-1}, \\
\Phi_{00} &= \theta(v) q^2 X^{-1}, \\
\Phi_{02} &= \theta(u)\theta(v) q^2 X^{-1} \left(ER\right)^{-1/2} \left[\sqrt{\Delta\delta}\left(B + a_0 A\right) + i\left(\Delta A - a_0 \delta B\right)\right].
\end{aligned}
\tag{7}
$$

We recall that exact determination of the phases (f, g) of the Maxwell spinors $\Phi_2 = qX^{-1/2}e^{if}$ and $\Phi_0 = qX^{-1/2}e^{ig}$ is possible through the tedious integration of the Maxwell equations. Fourtunately this is not necessary in the present study and for this reason we shall ignore it.

3.1 Initial Data and The Boundary Conditions.

→ The initial data associated with Region II $(u \geq 0, v < 0)$ is obtained by dropping the v in the metric. This amounts to the substitution $\tau = \sigma = \sin(u\theta\,(u))$, so that the metric functions take the form

$$\begin{aligned} R(u) &= \left(1 + a_0^2\right)\cos^2 u, \\ X(u) &= (p + \sin u)^2 + (l_0 + a_0 \sin u)^2\,, \\ E(u) &= \cos^2 u \left[\left(a_0 \cos^2 u - 2l_0 \sin u\right)^2 + \left((p + \sin u)^2 + a_0^2 + l_0^2\right)^2\right], \\ G(u) &= \cos^2 u \left[a_0 \cos^2 u - 2l_0 \sin u + a_0 \left((p + \sin u)^2 + a_0^2 + l_0^2\right)\right], \end{aligned} \tag{8}$$

→ The only non zero Ricci component in this region is given by

$$\Phi_{22}(u) = \theta(u) q^2 X^{-1}(u), \tag{9}$$

while the gravitational wave component $\Psi_4\,(u)$ has the form

$$\Psi_4\,(u) = (\textit{constant})\,\delta\,(u) + \theta\,(u)\,L\,(u)\,, \tag{10}$$

→ Similar data is obtained for region III $(u < 0, v \geq 0)$ by the substitution $\tau = -\sigma = \sin(v\theta\,(v))$. Obviously region III has the corresponding non-vanishing Ricci and Weyl components

$$\begin{aligned}\Phi_{00}(v) &= \theta(v) q^2 X^{-1}(v), \qquad (11)\\ \Psi_0(v) &= (\textit{constant})\,\delta(v) + \theta(v)\,K(v)\,,\end{aligned}$$

where $\delta(u)$ stands for the Dirac delta function, $L(u)$ and $K(v)$ are well defined function whose exact form is not of much interest here and we shall not give them explicitly.

→ Further extrapolation, by letting $u < 0$ and $v < 0$ in (3) reduces our metric into

$$ds^2 = 4\left(p^2 + l_0^2\right) dudv - (dx - a_0 dy)^2 - (cdy - a_0 dx)^2 \ , \qquad (12)$$

where

$$c = p^2 + a_0^2 + l_0^2,$$

which is manifestly flat in a rescaled coordinate system.

$\rightarrow$ In the presence of electromagnetic waves it had been shown that the appropriate boundary conditions are those of O'Brien and Synge. For more detail on the choice of the boundary conditions we refer to the book written by Jerry Griffths (Colliding Plane Waves in General Relativity, Oxford Press, 1990 and references cited therein). The critical condition to be checked is the continuity of

$$g^{ij}g_{ij,u}, \qquad \text{on} \qquad u = 0$$

and

$$g^{ij}g_{ij,v}, \qquad \text{on} \qquad v = 0.$$

When worked out in detail these reduce to the requirements that $(\ln \Delta\delta)_{,u}$ and $(\ln \Delta\delta)_{,v}$ are both continuous across $u = 0$ and $v = 0$, respectively. In summary, these are both continuous, the O'Brien - Synge conditions are satisfied and no extra sources are created in the collision process derived from the isometry with the KNN black hole.

3.2 The Weyl and Ricci Scalars.

→ Our interaction region $(u > 0, v > 0)$, metric (3) is equivalent (isometric) to the KNN metric (1), in fact it is obtained from the latter by a coordinate transformation. In this section we wish to make use of this advantage to find a proper tetrad that gives Ψ_2 and Φ_{11} as the only nonvanishing Weyl and Ricci scalars.

→ Our choice of the proper tetrad is,

$$\begin{aligned} l^\mu &= \left(k, 0, -\frac{D}{\Delta}, -\frac{a}{\Delta}\right), \\ 2n^\mu &= \frac{1}{k^2 Z}\left(k\Delta, 0, D, a\right), \\ \sqrt{2}m^\mu &= \frac{1}{\sqrt{\delta}\left(l + a\sigma - i(1+k\tau)\right)}\left(0, i\delta, a\delta - 2l\sigma, 1\right), \end{aligned} \tag{13}$$

where

$$\begin{aligned} Z &= (1+k\tau)^2 + (l+a\sigma)^2, \\ D &= (1+k\tau)^2 + l^2 + a^2, \\ k^2 &= 1 - a^2 + l^2 - Q^2, \end{aligned}$$

in which the parameter k is related to our previous parameter p through

$$p = \frac{1}{k}.$$

$\rightarrow$ In this proper tetrad the type-D character of our space-time becomes manifest with the Ψ_2 and Φ_{11} as follows

$$\Psi_2 = -\frac{1 - il}{\left[1 + k\tau - i\left(l + a\sigma\right)\right]^3}, \tag{14}$$

$$\Phi_{11} = \frac{Q^2}{2\left[\left(l + a\sigma\right)^2 + \left(1 + k\tau\right)^2\right]^2}. \tag{15}$$

$\rightarrow$ When $Q = 0$, the solution reduces to Kerr - NUT, which is the NUT extension of the paper I.
$\rightarrow$ When $Q = l = 0$, our solution reduces to the vacuum (i.e. Kerr) Einstein solution which corresponds to the Paper I.
$\rightarrow$ If we further choose $a = 0$ but $Q \neq 0$ and $l \neq 0$, the resulting metric corresponds to the charged - NUT metric which has not been considered by CX.
$\rightarrow$ The term $l + a\sigma$ in the Weyl scalar Ψ_2 represent the twist parameter of the gravitational wave. Hence, the NUT parameter l has the tendency to increase the existing twist in the Paper I and II.

4 Analytic Extension of the Space-Time Across the Horizon.

$\rightarrow$ The determinant of the metric in the u, v, x, y coordinates is given by,

$$\mid g \mid = X^2 \Delta \delta. \tag{16}$$

$\rightarrow$ Determinant vanishes on the surface $\tau = 1$, since $\Delta = 1 - \tau^2$.
$\rightarrow$ In order to perform the analytic extension, we shall adopt the method given by CX in Paper I. As a requirement of the method, at least one of the Killing vector fields should become null on the horizon. This is provided by calculating the norm of the Killing vectors on the horizon surface. The norm of the Killing vectors are

$$\begin{aligned} \mid \partial_x \mid^2 = g_{xx} &= -\left((p+\tau)^2 + (l_0 + a_0\sigma)^2 \right)^{-1} \left(\Delta + a_0^2 \delta \right), \\ \mid \partial_y \mid^2 = g_{yy} &= -\left((p+\tau)^2 + (l_0 + a_0\sigma)^2 \right)^{-1} \left(\Delta A^2 + \delta B^2 \right). \end{aligned} \tag{17}$$

$\rightarrow$ We observe that none of the norms of the Killing vectors vanish on the horizon surface (as $\tau \rightarrow 1$). In other words the Killing vectors ∂_x and ∂_y are both space-like.
$\rightarrow$ We choose the new Killing vector as,

$$\xi_1^\mu = \alpha\xi_x^\mu + \beta\xi_y^\mu, \tag{18}$$

where α and β are non-zero constants. We impose the condition that the norm of the new Killing vector $\mid \xi_1^\mu \mid^2$should vanish on the horizon surface as $\tau \to 1$.

$$\mid \xi_1^\mu \mid^2 = g_{\mu\nu}\left(\alpha\xi_x^\mu + \beta\xi_y^\mu\right)\left(\alpha\xi_x^\nu + \beta\xi_y^\nu\right) = 0.$$

Hence, the new Killing vector is obtained as,

$$\xi_1^\mu = \delta_x^\mu + c_1\delta_y^\mu, \tag{19}$$

where $c_1 = \frac{\beta}{\alpha} = \frac{a_0}{X_0}$ with $X_0 = (1+p)^2 + a_0^2 + l_0^2$. Note that the major difference in our choice is that, the new Killing vector lies in the xy - plane ($x^1 = x$ and $x^2 = y$), whereas, in CX case the Killing vector was becoming null on the x^2 ($= y$) axis.

→ At this stage we define two new coordinates as

$$\begin{aligned} \overline{x} &= x + c_1 y, \\ \overline{y} &= y - c_1 x. \end{aligned} \tag{20}$$

$\rightarrow$ In terms of the null coordinates, the metric (3) becomes

$$ds^2 = 2X du dv - \frac{X^{-1}X_0^2}{\left(X_0^2+a_0^2\right)^2}\{\left[X_0^2 R + a_0^2 E - 2GX_0 a_0\right] d\overline{x}^2 + \left[a_0^2 R + X_0^2 E + 2GX_0 a_0\right] d\overline{y}^2 - 2\left[X_0 a_0 \left(R-E\right) + \left(X_0^2 - a_0^2\right) G\right] d\overline{x} d\overline{y}\} \tag{21}$$

$\rightarrow$ The norm of the new Killing vector $\partial_{\overline{x}}$ and its scalar product with the other Killing vector $\partial_{\overline{y}}$ are given by,

$$\mid \partial_{\overline{x}} \mid^2 = -\frac{X^{-1}X_0^2}{X_0^2+a_0^2}\left\{\left[X + (1-\tau)(2p+1+\tau)\right]^2 + \frac{a_0^2\delta(1-\tau)}{(1+\tau)}(2p+1+\tau)^2\right\}\Delta \tag{22}$$

and

$$(\partial_{\overline{x}} \cdot \partial_{\overline{y}}) = 2\frac{X^{-1}X_0^2}{X_0^2+a_0^2}\{X_0 a_0 + A\left\{X_0\left[X + (1-\tau)(2p+1+\tau)\right] - a_0^2\right\} + \frac{a_0\delta(2p+1+\tau)}{1+\tau}\left\{X_0 B + a_0^2\right\}\}\Delta. \tag{23}$$

$\rightarrow$ It is clear to observe that both vanish in the limit $\tau \rightarrow 1$. We rewrite the metric (20) in terms of the new variables $\widetilde{l}$ and r defined by

$$\begin{aligned}\widetilde{l} &= \sqrt{\Delta\delta} = 1 - \sin^2 u - \sin^2 v, \\ r &= \tau\sigma = \cos^2 v - \cos^2 u.\end{aligned} \tag{24}$$

$\rightarrow$ This transforms metric (20) into

$$\begin{aligned} ds^2 = {} & \frac{X}{H}\left(d\widetilde{l}^2 - dr^2\right) - \frac{X^{-1}X_0^2}{(X_0^2+a_0^2)^2}\{\left[X_0^2 R + a_0^2 E - 2G X_0 a_0\right] d\overline{x}^2 \\ & + \left[a_0^2 R + X_0^2 E + 2G X_0 a_0\right] d\overline{y}^2 \\ & - 2\left[X_0 a_0 \left(R - E\right) + \left(X_0^2 - a_0^2\right) G\right] d\overline{x} d\overline{y}\}, \end{aligned} \tag{25}$$

where $H = \delta - \Delta = \sin 2u \sin 2v$. It can be shown easily that the coordinate singularity at $\tau = 1$ is removed when we apply the following transformation,

$$\xi = \widetilde{l}e^{c\overline{x}} \quad \text{and} \quad \zeta = \widetilde{l}e^{-c\overline{x}}, \tag{26}$$

in which $c = \frac{X_0}{X_0^2 + a_0^2}$ and in terms of the new coordinates (ξ, ζ), we have

$$\tau = \frac{1}{2}\left\{\sqrt{(1+r)^2 - \xi\zeta} + \sqrt{(1-r)^2 - \xi\zeta}\right\}, \tag{27}$$
$$\sigma = \frac{1}{2}\left\{\sqrt{(1+r)^2 - \xi\zeta} - \sqrt{(1-r)^2 - \xi\zeta}\right\},$$
$$H = \sqrt{(1+\xi\zeta - r^2)^2 - 4\xi\zeta}.$$

→ The exact metric (20) can be written in such a way that the absence of any singularity when $\xi = 0$ and/or $\zeta = 0$ is manifest, as follows

$$ds^2 = \frac{1}{2HX\delta}\left\{\overline{A}\left(\zeta^2 d\xi^2 + \xi^2 d\zeta^2\right) + \overline{B} d\xi d\zeta\right\} + C\left(\zeta d\xi - \xi d\zeta\right) d\overline{y} - \frac{X}{H} dr^2 - D d\overline{y}^2, \tag{28}$$

where

$$\overline{A} = \frac{1}{2\delta}\left(X^2 - \frac{\Sigma}{(1+\tau)^2}\right), \qquad \overline{B} = X^2\left(2\delta - \Delta\right) + \frac{1-\tau}{1+\tau}\Sigma,$$
$$C = \frac{X_0 \widetilde{\Sigma}}{X\left(X_0^2 + a_0^2\right)\delta}, \qquad D = \frac{X_0^2 \widetilde{\widetilde{\Sigma}}}{X\left(X_0^2 + a_0^2\right)^2},$$

with

$$\begin{aligned}\Sigma &= H\left(2p+1+\tau\right)\left[2X(1+\tau)+\left(2p+1+\tau\right)R\right],\\ \widetilde{\Sigma} &= a_0X_0 + A\left\{X_0\left[X+\left(1-\tau\right)\left(2p+1+\tau\right)\right]-a_0^2\right\}+\\ &\frac{a_0\delta\left(2p+1+\tau\right)}{1+\tau}\left\{X_0B+a_0^2\right\},\\ \widetilde{\widetilde{\Sigma}} &= \Delta\left[a_0+AX_0\right]^2+\delta\left\{a_0^2+X_0B\right\}^2.\end{aligned}$$

→ The determinant of the metric in the extended domain is expressed in the form,

$$\mid g \mid = \frac{X}{4H}\left\{2\overline{A}\left(2\overline{A}D+C^2\right)\left(\xi\zeta\right)^2+\overline{B}\left(C^2\xi\zeta-\overline{B}D\right)\right\}. \quad (29)$$

→ In the limit as $\xi=0$ and/or $\zeta=0$ (which is equivalent to $\tau=1$), the determinant becomes:

$$\mid g \mid = -\left[\left(p+1\right)^2+\left(l_0+a_0\sigma\right)^2\right]^4 X_0^2\delta^2.$$

Note that the above determinant vanishes when $\sigma=\pm1$. The points $\sigma=\pm1$, however, correspond to the null boundaries separating the interaction region from the incoming regions.

→ In other words, the metric (27) represents only the extension of the interaction region.

→ The contravariant components of the metric (27) are

$$g^{\xi\xi} = \frac{\xi^2\left(4\widetilde{A}D + C^2\right)}{\left(2\widetilde{A}\xi\zeta + \widetilde{B}\right)\left[\left(2\widetilde{A}D + C^2\right)\xi\zeta - \widetilde{B}D\right]}, \qquad g^{\zeta\zeta} = \frac{\zeta^2\left(4\widetilde{A}D + C^2\right)}{\left(2\widetilde{A}\xi\zeta + \widetilde{B}\right)\left[\left(2\widetilde{A}D + C^2\right)\xi\zeta - \widetilde{B}D\right]} \quad (30)$$

with

$$\widetilde{A} = \frac{\overline{A}}{2HX\delta}, \qquad \widetilde{B} = \frac{\overline{B}}{2HX\delta}.$$

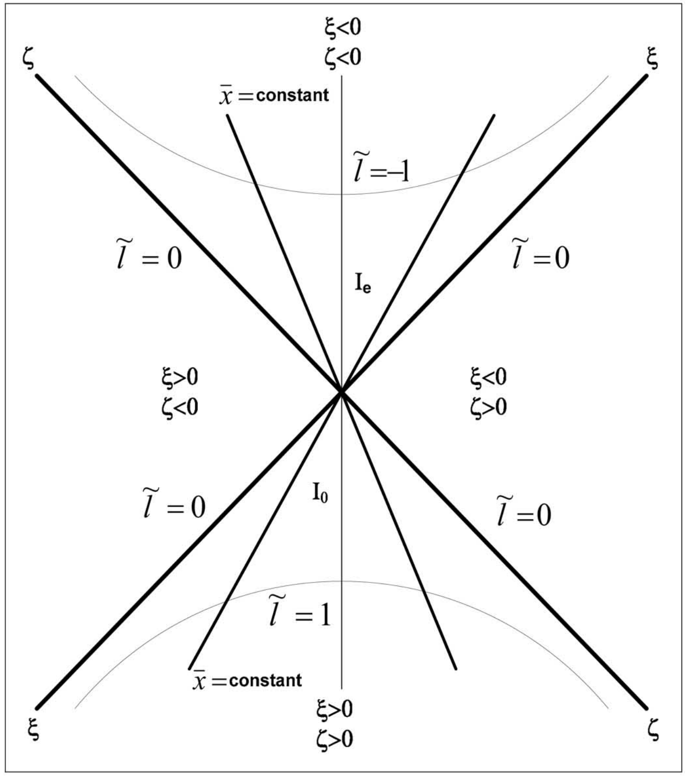
→ The nature of the surface when $\xi = \zeta = 0$ is identified by calculating the squared norms of the vector fields orthogonal to the surfaces $\xi_0 =$ *constant* and $\zeta_0 =$ *constant*. Let the surface $S(\xi) = \xi - \xi_0$ be such that $\xi = \xi_0$ is not a singular surface of the metric. The normal vector N^μ to the surface is defined by $N^\mu = g^{\mu\nu}S_{,\nu}$. Similarly for a surface $\zeta = \zeta_0$, the norm squares are then obtained by

$$N^2 = (\nabla S)^2 = g^{\mu\nu}\partial_\mu S \partial_\nu S = g^{\xi\xi}, \;\; N^2 = (\nabla S)^2 = g^{\mu\nu}\partial_\mu S \partial_\nu S = g^{\zeta\zeta}. \quad (31)$$

It is clear to observe that the vector fields $\partial_\mu \xi$ and $\partial_\mu \zeta$ become null on the hypersurface $\xi = 0, \;\; \zeta = 0$ respectively. Therefore the surface

$$\xi\zeta = \Delta\delta = 0 \quad (32)$$

consists of two null surfaces as depicted in figure below.



There are four distinct regions assigned to the coordinates $(\xi, \zeta, r, \overline{y})$. These regions are ; (i) $\xi > 0$, $\zeta > 0$ which is part of the interaction region I_0; (ii) $\xi < 0$, $\zeta < 0$ isometric region to I_0, since the simultaneous change in the sign of ξ and ζ leaves the metric (27) invariant; (iii) the region for $\xi < 0$, $\zeta > 0$ and (iv) for $\xi > 0$, $\zeta < 0$.

$\rightarrow$ From the curvature scalars as given in equations (13 and 14), the only possibility that the curvature singularity may develop when,

$$l + a\sigma = 0 \qquad \text{and} \qquad 1 + k\tau = 0 \tag{33}$$

occur simultaneously.

$\rightarrow$ This leads to $\tau = -\frac{1}{k}$ which is physically not acceptable since the range of τ is $0 \leq \tau \leq 1$, which is positive definite. Even if $k < 0$ is chosen (i.e. negative root from Eq.(12)) we can use the freedom of the NUT parameter l, so that, for $l > a$ implies $l + a\sigma \neq 0$, because the range of σ is $-1 \leq \sigma \leq 1$ which makes $\mid g \mid \neq 0$. This is in marked distinction from the case of CX for which $\mid g \mid = 0$, in the extended domain. Thus, the determinant of the metric together with the curvature scalars remain bounded in the regions when $\xi < 0$, $\zeta > 0$ or $\xi > 0$, $\zeta < 0$. As a result, the extended domain is free of any kind of singularity.

$\rightarrow$ The hypersurface $\xi = 0$ $(\zeta = 0)$ is a null surface and behaves like the event horizon of a black hole (one way membrane). This implies that the future directed time-like or null trajectories originating from region I_0 can enter the region $\xi > 0$, $\zeta < 0$ or $\xi < 0$, $\zeta > 0$ and from these regions to the isometric region I_e.

5 CONCLUSION.

The physical significance of the NUT parameter in the theory of relativity has been one of the discussion subject that its exact physical interpretation is not clarified yet. In this paper, we have analysed the effect of the NUT parameter in the context of CGW spacetimes. In our analysis we considered the NUT extension of Paper II which is locally isometric to the part of a region in between the inner and outer horizons of KNN BH. Our main concern in this article is not only to provide a new CGW geometry but also to explore the physical effect of the NUT parameter. We have shown that the NUT parameter provides an additional twist to gravitational waves. As a result, the waves that participate in the collision is modified with respect to the cases in Paper I and II. The overall effect of this modification becomes more clear when a non-unique extension beyond the Cauchy horizon is obtained. The initial data of CX (in Paper I and II) , however, was able to transform the coordinate singularity into a harmless time-like singularity in the extended domain. In our case, we have shown that the modification in the initial data as a result of the inclusion of the NUT parameter removes the time-like singularities as well and leaves the extended spacetime singularity free.

To conclude the paper we wish to express the view that the CX duality is more than a mathematical equivalence: It must address deeper implications concerning BHs and colliding waves that awaits yet to be explored.