

Radiative Corrections in the Electroweak Sector of the Standard Model Extension (SME)

Don Colladay

New College of Florida

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- DC and P. McDonald

Overview of Talk

- Introduction to Functional Determinants
- Electroweak Sector SME, and Lorentz Violation
- Higgs Sector corrections
- Fermion Sector corrections

Introduction to Functional Determinants

Basic Gaussian integral is of form

$$\int dx e^{-\alpha x^2} = \sqrt{\frac{\pi}{\alpha}}$$

Can be easily generalized to N dimensional matrix \mathbf{A} in exponent with integration over N-component vector \vec{x}

$$\int d^N x e^{-\vec{x} \cdot \mathbf{A} \cdot \vec{x}} = C \frac{1}{\sqrt{\det(\mathbf{A})}}$$

where C is a constant.

This expression can be used to construct the "definition" of a functional determinant of a linear operator S

$$\frac{1}{\sqrt{\det(S)}} = C \int \mathcal{D}\phi e^{-\langle \phi | S | \phi \rangle}$$

⇒ Integrals appear in path integral approach to field theory

(Subtlety: integrals are divergent and need to be regularized...)

Example application to scalar ϕ^4 field theory with action

$$S = \int d^4x \mathcal{L} = \int d^4x \left[\frac{1}{2} \partial_\mu \phi \partial^\mu \phi - \frac{m^2}{2} \phi^2 + \frac{\lambda}{4!} \phi^4 \right]$$

leads to generating functional for connected Green functions

$$Z[j] = \int \mathcal{D}\phi \exp \left[iS + i \int d^4x \phi(x) j(x) \right]$$

(the measure is normalized such that $Z[0] = 1$)

Classical (extremal action)solution solves the equation of motion

$$(\partial_\mu \partial^\mu + m^2 - \frac{\lambda}{3!} \phi_{cl}^2) \phi_{cl} = j \phi_{cl}$$

Can expand field in terms of small fluctuations about classical action

$$\phi(x) \rightarrow \phi_{cl} + \phi$$

Generating functional in saddle point evaluation to $\mathcal{O}(\phi^2)$

$$Z[j] = e^{iS_{cl}[\phi_{cl}]} \int \mathcal{D}\phi \exp \left(-i \int d^4x \left[\frac{1}{2} \phi \left\{ \partial_\mu \partial^\mu + m^2 - \frac{\lambda}{2} \phi_{cl}^2 \right\} \phi \right] \right)$$

yielding functional determinant of $\partial_\mu \partial^\mu + m^2 - \frac{\lambda}{2} \phi_{cl}^2$

Evaluation of determinant

$$\text{Det} \left[\partial_\mu \partial^\mu + m^2 - \frac{\lambda}{2} \phi_{cl}^2 \right] = \text{Det} \left[P(1 + P^{-1} \Delta) \right]$$

with $P = \partial_\mu \partial^\mu + m^2$ denoting free field piece and $\Delta = -\frac{\lambda}{2} \phi_{cl}^2$
($\text{Det} P$ piece drops out from normalization)

Useful identity

$$\ln \text{Det} S = \text{Tr} \ln S$$

converts calculation to

$$\text{Tr} \ln(1 + P^{-1} \Delta) = \text{Tr} \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} (P^{-1} \Delta)^n$$

$$P^{-1}(x - y) = \int \frac{d^4 p}{(2\pi)^4} \frac{1}{p^2 - m^2} e^{ip \cdot (x-y)}$$

$n = 1$ term

$$\text{Tr}(P^{-1}\Delta) = \frac{\lambda}{2} \left[\int \frac{d^4 p}{(2\pi)^4} \frac{1}{p^2 - m^2 + i\epsilon} \right] \int d^4 x \phi_{cl}^2(x)$$

Regularize integral (dimensional regularization)

$$\int \frac{d^d p}{(2\pi)^d} \left(\frac{1}{p^2 - m^2 + i\epsilon} \right) = -\frac{i\Gamma(1 - d/2)}{(4\pi)^{d/2}} \frac{1}{(m^2)^{1-d/2}} \rightarrow \frac{im^2}{8\pi^2\epsilon}$$

has simple pole as $d \rightarrow 4$ ($d = 4 - \epsilon$) and

$$\text{Tr}(P^{-1}\Delta) = \frac{i\lambda m^2}{16\pi^2\epsilon}$$

Comparing this term with the classical action term

$$\ln Z = iS_{cl} - \frac{1}{2} \ln \text{Det}(P^{-1}\Delta) = im^2 \left(1 - \frac{\lambda}{32\pi^2\epsilon}\right) \int d^4x \phi_{cl}^2$$

Makes it immediately obvious that the pole can be absorbed into the mass and gives the appropriate multiplicative renormalization constant (valid to one-loop)

$$\Rightarrow \boxed{m_r = m \left(1 - \frac{\lambda}{32\pi^2\epsilon}\right)}$$

This procedure may be repeated for the next order ($n = 2$) expansion in the $\text{Tr} \ln(1 + P^{-1}\Delta)$ term

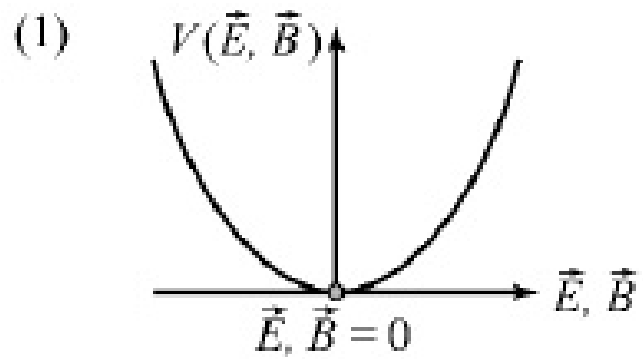
$$\Rightarrow \boxed{\lambda_r = \lambda \left(1 + \frac{6\lambda}{32\pi^2\epsilon}\right)}$$

Motivation and Implementation of Lorentz Violation

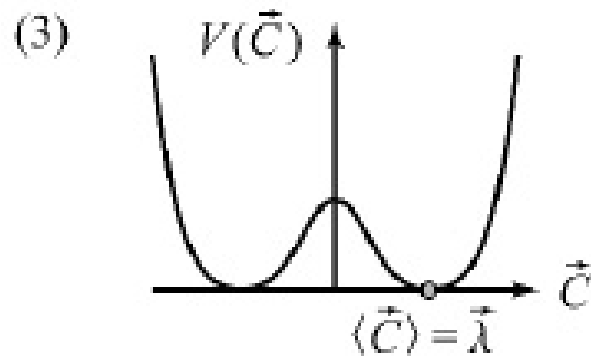
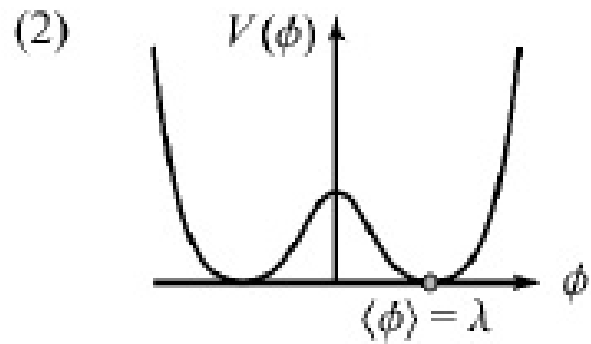
- Generic theories underlying standard model "naturally" allow for Lorentz-breaking effects: Original conception in string theory
 - Kostelecky and Samuel 1989
- Incorporate into standard model using effective field theory to generate Standard Model Extension (SME)
 - DC and Kostelecky 1997

⇒ advantage of model independent framework

One w



reaking



- Lehnert hep-ph/0611177

Introduction to Electroweak SME

Left- and Right-handed lepton multiplets are ($m_\nu = 0$)

$$L_A = \begin{pmatrix} \nu_A \\ l_A \end{pmatrix}_L, \quad R_A = (l_A)_R,$$

where

$$\psi_L \equiv \frac{1}{2}(1 - \gamma_5)\psi, \quad \psi_R \equiv \frac{1}{2}(1 + \gamma_5)\psi,$$

as usual, and where $A = 1, 2, 3$ labels the flavor: $l_A \equiv (e, \mu, \tau)$, $\nu_A \equiv (\nu_e, \nu_\mu, \nu_\tau)$. The conventional leptonic lagrangian is

$$\mathcal{L}_{\text{lepton}} = \frac{1}{2}i\bar{L}_A\gamma^\mu \overleftrightarrow{D}_\mu L_A + \frac{1}{2}i\bar{R}_A\gamma^\mu \overleftrightarrow{D}_\mu R_A.$$

SME leptonic corrections preserving gauge group $SU(2) \times U(1)$:

$$\mathcal{L}_{\text{lepton}}^{\text{CPT-even}} = \frac{1}{2}i(c_L)_{\mu\nu AB} \bar{L}_A \gamma^\mu \overleftrightarrow{D}^\nu L_B + \frac{1}{2}i(c_R)_{\mu\nu AB} \bar{R}_A \gamma^\mu \overleftrightarrow{D}^\nu R_B \quad ,$$

$$\mathcal{L}_{\text{lepton}}^{\text{CPT-odd}} = -(a_L)_{\mu AB} \bar{L}_A \gamma^\mu L_B - (a_R)_{\mu AB} \bar{R}_A \gamma^\mu R_B \quad ,$$

where $(c)_{\mu\nu AB}$ and $(a)_{\mu AB}$ are constant background couplings that allow for cross-couplings between generations.

Higgs sector standard model terms (ϕ is SU(2) doublet)

$$\mathcal{L}_{\text{Higgs}} = (D_\mu \phi)^\dagger D^\mu \phi + \mu^2 \phi^\dagger \phi - \frac{\lambda}{3!} (\phi^\dagger \phi)^2 \quad ,$$

SME corrections to Higgs

$$\begin{aligned} \mathcal{L}_{\text{Higgs}}^{\text{CPT-even}} = & \frac{1}{2} (k_{\phi\phi})^{\mu\nu} (D_\mu \phi)^\dagger D_\nu \phi + h.c. \\ & - \frac{1}{2} (k_{\phi B})^{\mu\nu} \phi^\dagger \phi B_{\mu\nu} - \frac{1}{2} (k_{\phi W})^{\mu\nu} \phi^\dagger W^{\mu\nu} \phi \quad , \end{aligned}$$

containing couplings to gauge bosons W^μ and B^μ and

$$\mathcal{L}_{\text{Higgs}}^{\text{CPT-odd}} = i (k_\phi)^\mu \phi^\dagger D^\mu \phi + h.c.$$

Yukawa couplings in standard case are

$$\mathcal{L}_{\text{Yukawa}} = -(G_L)_{AB} \bar{L}_A \phi R_B + \text{h.c.},$$

with SME corrections

$$\mathcal{L}_{\text{Yukawa}}^{\text{CPT-even}} = -\frac{1}{2} (H_L)_{\mu\nu AB} \bar{L}_A \phi \sigma^{\mu\nu} R_B + \text{h.c.}$$

where $\sigma^{\mu\nu} = \frac{i}{2} [\gamma^\mu, \gamma^\nu]$ as usual

SME gauge corrections

$$\mathcal{L}_{\text{CPT-even}} = -\frac{1}{2}(k_F)_{\mu\nu\alpha\beta} \text{tr} F^{\mu\nu} F^{\alpha\beta}$$

$$\mathcal{L}_{\text{CPT-odd}} = -\frac{1}{2}(k_{AF})^\kappa \epsilon_{\kappa\lambda\mu\nu} \text{tr} (A^\lambda F^{\mu\nu} - \frac{2}{3}ig A^\lambda A^\mu A^\nu)$$

where $(k_F)^{\mu\nu\alpha\beta}$ and $(k_{AF})^\kappa$ are constant background couplings

(A^μ can be either B^μ or W^μ SM gauge fields)

Some related work on renormalization in SME

- One Loop Renormalization of Lorentz-violating QED
Kostelecky, Lane, and Pickering, PRD 2002
- Lorentz-violating QED renormalization in curved space
Barredo-Peixoto and Shapiro, PLB 2006
- Lorentz violation and Faddeev-Popov Ghosts
Altschul, PRD 2006
- One-Loop Renorm of Lorentz-violating Pure Yang Mills
D. C., McDonald, PRD 2007
- One-Loop Renorm of QED with Lorentz Violation
D.C., McDonald, PRD 2008

Recent additional work relating to anomalies

- Chiral Anomaly Beyond Lorentz Invariance
Arias, et. al, PRD 2007
- Relaxing Lorentz Invariance in General Perturbative Anomalies
Salvio, PRD 2008

One-Loop Effective Action

$$\exp i\Gamma[\Psi] = \int \mathcal{D}\Psi e^{i \int \mathcal{L}(\Psi, \partial^\mu \Psi)}$$

where Ψ represents all dynamic fields in standard model
Expand fields about classical background field $\underline{\Psi}$

$$\Psi = \underline{\Psi} + \psi$$

retain quadratic terms in the fluctuation fields

→ Functional Determinants

CPT-even gauge field terms

Saddle point evaluation requires quadratic (in \mathcal{A}) contribution

$$\mathcal{L}_{\mathcal{A}}^{quad} = -\frac{1}{2g^2} \text{tr} \mathcal{A}^\mu \left[-g_{\mu\nu} \underline{D}^2 - 2i \underline{F}_{\mu\nu} - i(k_F)_{\alpha\beta\mu\nu} \underline{F}^{\alpha\beta} - 2(k_F)_{\mu\alpha\nu\beta} \underline{D}^\alpha \underline{D}^\beta \right] \mathcal{A}^\nu .$$

The trace is extended to cover Lorentz indices using notation

$$\begin{aligned} (\tau_{\alpha\beta})_{\mu\nu} &= i(g_{\alpha\mu}g_{\beta\nu} - g_{\alpha\beta}g_{\mu\nu}) \\ (k_F^I)_{\alpha\beta\mu\nu} &= (k_F)_{\alpha\beta\mu\nu} \\ (k_F^{II})_{\alpha\beta\mu\nu} &= (k_F)_{\mu\alpha\nu\beta} \end{aligned}$$

then

$$\mathcal{L}_{\mathcal{A}}^{quad} = -\frac{1}{2g^2} \text{tr} \mathcal{A} (\Delta_A) \mathcal{A}$$

Contributes $Det\Delta_A$ to effective action

$$\Delta_A = P_A + \Delta_A^{(1)} + \Delta_A^{(2)} + \Delta_A^{(F)}$$

$$P_A = -(g_{\alpha\beta} + 2k_{F\alpha\beta}^{II})\partial^\alpha\partial^\beta$$

$$\Delta_A^{(1)} = -i(g_{\alpha\beta} + 2k_{F\alpha\beta}^{II})(\partial^\alpha \underline{A}^\beta + \underline{A}^\alpha \partial^\beta)$$

$$\Delta_A^{(2)} = (g_{\alpha\beta} + 2k_{F\alpha\beta}^I)(\underline{A}^\alpha \underline{A}^\beta)$$

$$\Delta_A^{(F)} = -[\tau_{\alpha\beta} + ik_{F\alpha\beta}^I]\underline{F}^{\alpha\beta}$$

The first term P_A is independent of \underline{A} and factors out

$$\ln Det(P_A^{-1}\Delta_A) = \ln Det \left[1 + P_A^{-1}(\Delta_A^{(1)} + \Delta_A^{(2)} + \Delta_A^{(F)}) \right].$$

Using the relationship

$$\ln \text{Det} S = \text{Tr} \ln S$$

and expanding the \ln to second-order gives

$$\ln \text{Det}(P_A^{-1} \Delta_A) = \frac{i}{(4\pi)^2} C_2(G) \Gamma(2 - \frac{d}{2}) \otimes \int \frac{d^4 k}{(2\pi)^4} \left[\left(\frac{7}{3}\right) \kappa_{F\mu\lambda\nu}{}^\lambda Q^{\mu\nu} - (12) \kappa_{F\mu\alpha\nu\beta} (k^\alpha k^\beta \underline{A}^\mu \underline{A}^\nu) \right].$$

where

$$Q^{\mu\nu} = (k^\mu k^\nu \underline{A}^2 - 2k^\mu \underline{A}^\nu k \cdot \underline{A} + k^2 \underline{A}^\mu \underline{A}^\nu)$$

Addition of the corresponding term in classical effective action

$$e^{i\Gamma[A]} = e^{iS_{cl}[A]} (\text{Det}\Delta_A)^{-1/2}$$

yields one-loop divergent contribution to (trace-free) k_F coupling

$$\mathcal{L}_{eff} \supset -\frac{1}{4g^2} \left(1 - \frac{6g^2}{(4\pi)^2} \Gamma\left(2 - \frac{d}{2}\right) \right) (k_F)_{\mu\alpha\nu\beta} \underline{F}^{\mu\nu} \underline{F}^{\alpha\beta}$$

Allows immediate identification of renormalization parameters

$$\begin{aligned} g_b &= Z_g g_r \\ (k_F)_b &= Z_{k_F} (k_F)_r \end{aligned}$$

$$Z_g = 1 - (11/6) \frac{g^2 C_2(G)}{(4\pi)^2} \Gamma\left(2 - \frac{d}{2}\right). \quad (1)$$

$$\boxed{Z_{k_F} = 1 + (7/3) \frac{g^2 C_2(G)}{(4\pi)^2} \Gamma\left(2 - \frac{d}{2}\right)} \quad (2)$$

Trace k_F terms take the form

$$k_F^{\mu\nu\alpha\beta} = \Lambda^{\mu\alpha} g^{\nu\beta} - \Lambda^{\nu\alpha} g^{\mu\beta} - \Lambda^{\mu\beta} g^{\nu\alpha} + \Lambda^{\nu\beta} g^{\mu\alpha}$$

where $\Lambda^{\mu\nu}$ is symmetric matrix

Comparison to classical term gives

$$\mathcal{L}_{eff} \supset -\frac{1}{g^2} \left(1 - \frac{11 C_2(G) g^2}{3 \cdot 4\pi^2} \Gamma\left(2 - \frac{d}{2}\right) \right) \Lambda^{\mu\nu} \int \frac{d^4 k}{(2\pi)^4} Q^{\mu\nu}$$

where again

$$Q^{\mu\nu} = (k^\mu k^\nu \underline{A}^2 - 2k^\mu \underline{A}^\nu k \cdot \underline{A} + k^2 \underline{A}^\mu \underline{A}^\nu)$$

Divergence already absorbed using $g_b = Z_g g_r$

⇒ No renormalization of $\Lambda^{\mu\nu}$ required (without fermions)

CPT-odd gauge field terms

form of $\text{Det}\Delta_A$ contribution to effective action

$$\Delta_A = P_A + \Delta_A^{(1)} + \Delta_A^{(2)} + \Delta_A^{(F)}$$

$$P_A = -(g_{\alpha\beta} + ((k_{AF})^\kappa \epsilon_{\kappa\beta})) \partial^\alpha \partial^\beta$$

$$\Delta_A^{(1)} = -ig_{\alpha\beta} (\partial^\alpha \underline{A}^\beta + \underline{A}^\alpha \partial^\beta) - i(k_{AF})^\alpha \epsilon_{\alpha\beta} \underline{A}^\beta$$

$$\Delta_A^{(2)} = g_{\alpha\beta} \underline{A}^\alpha \underline{A}^\beta$$

$$\Delta_A^{(F)} = -\tau_{\alpha\beta} \underline{F}^{\alpha\beta}$$

where $\epsilon_{\alpha\beta}$ is matrix notation for $(\epsilon_{\alpha\beta})_{\mu\nu} = \epsilon_{\alpha\beta\mu\nu}$

$\Rightarrow k_{AF}$ contributions to functional determinant cancel!

$$\boxed{Z_g^2 = Z_{k_{AF}}}$$

Higgs sector determinants

$$\begin{aligned} \mathcal{L}_{\text{Higgs}}^{\text{CPT-even}} = & \frac{1}{2} (k_{\phi\phi})^{\mu\nu} (D_\mu\phi)^\dagger D_\nu\phi + h.c. \\ & - \frac{1}{2} (k_{\phi B})^{\mu\nu} \phi^\dagger \phi B_{\mu\nu} - \frac{1}{2} (k_{\phi W})^{\mu\nu} \phi^\dagger W^{\mu\nu} \phi , \end{aligned}$$

and

$$\mathcal{L}_{\text{Higgs}}^{\text{CPT-odd}} = i(k_\phi)^\mu \phi^\dagger D^\mu \phi + h.c.$$

where k_ϕ , $k_{\phi\phi}$, $k_{\phi B}$, and $k_{\phi W}$ are the various LV couplings. Note that they are all independent parameters in the generic SME.

$k_{\phi\phi}$ contributions

Expand about the background field as usual

$$\phi \rightarrow \underline{\phi} + \phi.$$

Yields the quadratic contribution

$$\mathcal{L}_{higgs} = \phi^\dagger \left[-D^2 + \mu^2 - \frac{1}{2} \left[(k_{\phi\phi})^{\mu\nu} D_\mu D_\nu + h.c. \right] \right] \phi$$

Integration over the higgs gives logdet of bracketed operator

To facilitate calculation, $k_{\phi\phi} = k_{\phi\phi}^R + ik_{\phi\phi}^I$ is split into a symmetric real part and an antisymmetric imaginary piece.

The kinetic, field-independent piece of the operator is

$$P = -(\eta^{\mu\nu} + k_{\phi\phi}^{\mu\nu})\partial_\mu\partial_\nu + \mu^2 ,$$

The inverse of this operator is written using the Fourier expansion

$$P^{-1} = \int \frac{d^4p e^{-ip\cdot(x-y)}}{(2\pi)^4 \left((\eta^{\mu\nu} + k_{\phi\phi}^{\mu\nu})p_\mu p_\nu + \mu^2 \right)} .$$

The contribution of the real part of $k_{\phi\phi}$ to logdet is

$$\logdet \left[-D^2 - \frac{1}{2} (k_{\phi\phi}^R)^{\mu\nu} D_\mu D_\nu \right] =$$

$$\frac{1}{6} \frac{i}{(4\pi)^2} \Gamma(2 - \frac{d}{2}) (k_{\phi\phi}^R)^{\mu\nu} Q^{\mu\nu}$$

where the gauge invariant operator $Q^{\mu\nu}$ is defined as before

$$Q^{\mu\nu} = tr \int \frac{d^4k}{(2\pi)^4} (k^\mu k^\nu \underline{A}^2 - 2k^\mu \underline{A}^\nu k \cdot \underline{A} + k^2 \underline{A}^\mu \underline{A}^\nu) . \quad (3)$$

Note that this term contributes radiative corrections to the anti-self-dual components of $(k_W)^{\mu\nu\alpha\beta}$ and $(k_B)^{\mu\nu\alpha\beta}$ and can be absorbed into the corresponding Z factors. ($\underline{A}^\mu = \frac{g}{2} W^\mu + \frac{g'}{2} B^\mu$)

The contribution of the imaginary part of $k_{\phi\phi}$ to logdet is

$$\log\det[-D^2 - \frac{1}{2}(k_{\phi\phi}^I)^{\mu\nu} D_\mu D_\nu] =$$
$$-\frac{g'}{4} \int \frac{d^4p}{(2\pi)^4(p^2 + \mu^2)} (k_{\phi\phi}^I)^{\mu\nu} B_{\mu\nu}$$

yielding a quadratically divergent linear field instability

the corresponding contribution of $k_{\phi B}$ coupling is of same form

→ suggests setting $\frac{g'}{2}k_{\phi\phi}^I + k_{\phi B} = 0$ to obtain sensible theory which completely removes the $\phi^\dagger\phi B^{\mu\nu}$ coupling from the SME lagrangian at tree-level

Remaining higgs terms

$$-\frac{1}{2}(k_{\phi W})^{\mu\nu}\phi^\dagger W^{\mu\nu}\phi ,$$

gives zero contribution due to Lie Algebra trace,

$$\mathcal{L}_{\text{Higgs}}^{\text{CPT-odd}} = i(k_\phi)^\mu\phi^\dagger D^\mu\phi + h.c.$$

gives vanishing contribution due to field phase redefinition

$$\phi \rightarrow e^{i(k_\phi)^\mu x_\mu}\phi$$

that removes term from Lagrangian

Fermion Determinants

Leptonic piece of lagrangian is

$$\begin{aligned}\mathcal{L}_{\text{lepton}}^{\text{CPT-even}} &= \frac{1}{2}i(c_L)_{\mu\nu AB}\bar{L}_A\gamma^\mu\overleftrightarrow{D}^\nu L_B \\ &\quad + \frac{1}{2}i(c_R)_{\mu\nu AB}\bar{R}_A\gamma^\mu\overleftrightarrow{D}^\nu R_B \quad , \\ \mathcal{L}_{\text{lepton}}^{\text{CPT-odd}} &= -(a_L)_{\mu AB}\bar{L}_A\gamma^\mu L_B - (a_R)_{\mu AB}\bar{R}_A\gamma^\mu R_B \quad ,\end{aligned}$$

Integral over ψ gives functional determinant?

problem: degrees of freedom are ψ_L and ψ_R , not ψ

solution: collect chiral reps into multiplet of 2-component spinors

$$L = \begin{pmatrix} \nu_L \\ l_L \\ l_R \end{pmatrix}_A .$$

write standard lagrangian in matrix form $\mathcal{L} = L_A M_{AB} L_B$ with

$$M_{AB} = \begin{pmatrix} \frac{g}{2} \overline{\mathcal{W}}^3 - \frac{g'}{2} \overline{\mathcal{B}} & \frac{g}{\sqrt{2}} \overline{\mathcal{W}}^+ & (G_L)_{AB} \phi^+ \\ \frac{g}{\sqrt{2}} \overline{\mathcal{W}}^- & -\frac{g}{2} \overline{\mathcal{W}}^3 - \frac{g'}{2} \overline{\mathcal{B}} & (G_L)_{AB} \phi^0 \\ (G_L^\dagger)_{AB} \phi^- & (G_L^\dagger)_{AB} \phi^{*0} & -g' \overline{\mathcal{B}} \end{pmatrix},$$

where $\not{A} = \sigma^\mu A_\mu$ and $\overline{\not{A}} = \bar{\sigma}^\mu A_\mu$ in 2×2 sigma basis

Integration over chiral fields yields determinant with kinetic term

$$P_{AB} = \begin{pmatrix} (P_L)_{AB} & 0 & 0 \\ 0 & (P_L)_{AB} & 0 \\ 0 & 0 & (P_R)_{AB} \end{pmatrix},$$

where

$$(P_L)_{AB} = i(\not{\partial}\delta_{AB} + (c_L)_{\mu\nu AB}\bar{\sigma}^\mu\partial^\nu),$$

and

$$(P_R)_{AB} = i(\not{\partial}\delta_{AB} + (c_R)_{\mu\nu AB}\sigma^\mu\partial^\nu)$$

.

The contribution of the c -terms to the logdet in this case yields

$T = T_B + T_W$ where

$$T_B = -\frac{1}{3} \frac{i}{(4\pi)^2} g'^2 \Gamma(2 - \frac{d}{2}) \text{Tr}(c_L + 2c_R)_{\mu\nu} Q_B^{\mu\nu},$$

and

$$T_W = -\frac{1}{3} \frac{i}{(4\pi)^2} g^2 \Gamma(2 - \frac{d}{2}) \text{Tr}(c_L)_{\mu\nu} Q_W^{\mu\nu},$$

where $Q_B^{\mu\nu}$ and $Q_W^{\mu\nu}$ are defined as previously

$$Q^{\mu\nu} = \text{tr} \int \frac{d^4 k}{(2\pi)^4} (k^\mu k^\nu \underline{A}^2 - 2k^\mu \underline{A}^\nu k \cdot \underline{A} + k^2 \underline{A}^\mu \underline{A}^\nu). \quad (4)$$

with \underline{A} replaced with B or W respectively

→ Contributions absorbed by k_W and k_B terms of SME

Quark contributions

Quarks are similar to leptons with extra right-handed field
→ get 4-component multiplet

$$L_A = \begin{pmatrix} u_L \\ d_L \\ u_R \\ d_R \end{pmatrix}_A .$$

with corresponding 4×4 matrix M_{AB}

Determinant similar to leptonic case with different coupling factors to B field due to hypercharge assignments.

Ghost Violation

Can include extra Lorentz-breaking terms directly in ghost sector

$$\mathcal{L}_G = -\bar{c}(-\underline{D}^\mu \underline{D}_\mu - C_{\mu\nu} \underline{D}^\mu \underline{D}^\nu)c$$

yields additional functional determinant contribution

$$\log \det(P_c^{-1} \Delta_c) = -\frac{i}{6} \frac{C_2(G)}{(4\pi)^2} \Gamma(2 - \frac{d}{2}) C_{\mu\nu} \int \frac{d^4 k}{(2\pi)^4} Q^{\mu\nu}. \quad (5)$$

where again

$$Q^{\mu\nu} = (k^\mu k^\nu \underline{A}^2 - 2k^\mu \underline{A}^\nu k \cdot \underline{A} + k^2 \underline{A}^\mu \underline{A}^\nu)$$

\Rightarrow can be renormalized into Z_Λ (trace piece of k_F)

Summary

- Results indicate that previously unbounded third-generation $c^{\mu\nu}$ couplings can radiatively induce terms in the photon sector implying stringent bounds
- Conventional $SU(2) \times U(1)$ electroweak symmetry breaking is unmodified relative to tree-level case
- Together with previous calculations of the QCD sector, the functional determinants of the SME at one-loop can be absorbed in the standard way using multiplicative renormalization factors