Horizontal (flavor) Symmetry from Vertical (unified) Symmetry

S.M. Barr

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Reviving Two Old Ideas

- Flavor structure comes from grand unification gauge symmetry, not from family symmetry: i.e. fermions of different families transform differently under GUT group
- The large hierarchy among the masses of the different families is "radiative": i.e. some quarks and leptons get mass at tree level and some from loops.

The three families transform the same under the Standard Model group:

$$(3,2,\tfrac{1}{6})+(\overline{3},1,-\tfrac{2}{3})+(\overline{3},1,\tfrac{1}{3})+(1,2,-\tfrac{1}{2})+(1,1,+1)$$

And also under SU(5): $10+\overline{5}$

And under SO(10) (for the usual and simplest embedding): 16

However, under SU(N), with N>5, they typically come from different multiplets.

In SU(N), if the fermions are in antisymmetric tensor multiplets, no exotics appear: only $10+\overline{10}+\overline{5}+5+1$ arise in the SU(5) decomposition.

And a 10 can arise from different SU(N) multiplets: $\psi^{\alpha\beta}, \psi^{\alpha\beta\bullet}, \psi^{\alpha\beta\bullet\bullet}, \ldots$

As can a $\overline{5}$: $\psi_{\alpha}, \psi_{\alpha\bullet}, \psi_{\alpha\bullet\bullet}, \ldots$

AN SU(7) EXAMPLE: An Anomaly-Free Set that gives 3 FAMILIES is

$$[3]_L = \psi^{[ABC]} \rightarrow \psi^{\alpha\beta\gamma} + \psi^{\alpha\beta 6} + \psi^{\alpha\beta 7} + \psi^{\alpha 67}$$

$$35 \rightarrow \overline{10} + 10 + 10 + 5$$

$$2 \cdot [2]_L = \psi_{(a)}^{[AB]} \rightarrow \psi_{(a)}^{[\alpha\beta]} + \psi_{(a)}^{[\alpha 6]} + \psi_{(a)}^{[\alpha 7]} + \psi_{(a)}^{[67]} \qquad (a = 1, 2)$$

$$2(21) \rightarrow 2(10) + 2(5) + 2(5) + 2(1)$$

$$8 \cdot [\overline{1}] = \psi_{(m)A} \rightarrow \psi_{(m)\alpha} + \psi_{(m)6} + \psi_{(m)7} \qquad (m = 1, 2, \ldots, 8)$$

$$8(\overline{7}) \rightarrow 8(\overline{5}) + 8(1) + 8(1)$$

Thus the light families are contained in:

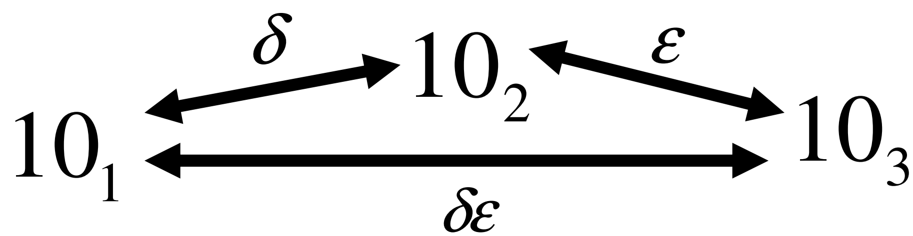
$\psi^{\alpha\beta}, \psi^{\alpha\beta 6}, \psi^{\alpha\beta 7} \rightarrow$ These 10 transform differently under SU(7)

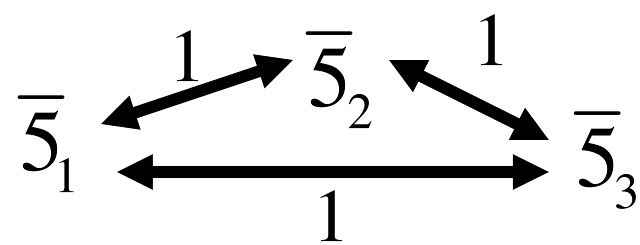
$\psi_{(1)\alpha}, \psi_{(2)\alpha}, \psi_{(3)\alpha} \rightarrow$ These $\overline{5}$ transform exactly the same under SU(7)

This is typical of simple SU(N) examples, as we'll see: Is this a drawback?

NO! It is actually very **GOOD** if the unified group distinguishes among the 10's of SU(5) but fails to distinguish among the $\bar{5}$'s !! It leads to the so-called "**LOPSIDED**" structure for the quark and lepton mass matrices:

SUPPOSE THAT TRANSITIONS AMONG THE 10's ARE SUPPRESSED BY THE GUT SYMMETRY THAT DISTINGUISHES THEM, BUT THAT TRANSITIONS AMONG THE $\bar{5}$'s ARE UNSUPPRESSED.





Recalling:

$$10 \supset \left(\ell^-_R, \begin{pmatrix} u_L \\ d_L \end{pmatrix}, u_R \right), \qquad \bar{5} \supset \left(\begin{pmatrix} \nu_L \\ \ell^-_L \end{pmatrix}, d_R \right)$$

Thus, there is large MNS mixing of LH leptons,

but small CKM mixing of LH quarks: $\theta_{23} \sim \varepsilon, \quad \theta_{12} \sim \delta, \quad \theta_{13} \sim \delta\varepsilon$

In fact, it even better than that (assume other families get mass from 3rd):

$$u_i (M_U)_{ij} u_j^c \sim 10_i \begin{bmatrix} \delta^2\varepsilon^2 & \delta\varepsilon^2 & \delta\varepsilon \\ \delta\varepsilon^2 & \varepsilon^2 & \varepsilon \\ \delta\varepsilon & \varepsilon & 1 \end{bmatrix} 10_j v_u \qquad \nu_i (M_\nu)_{ij} \nu_j \sim \bar{5}_i \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \bar{5}_j \frac{v_u^2}{M_R}$$

$$d_i (M_D)_{ij} d_j^c \sim 10_i \begin{bmatrix} \delta\varepsilon & \delta\varepsilon & \delta\varepsilon \\ \varepsilon & \varepsilon & \varepsilon \\ 1 & 1 & 1 \end{bmatrix} \bar{5}_j v_d \qquad \ell_i^- (M_L)_{ij} \ell_j^+ \sim \bar{5}_i \begin{bmatrix} \delta\varepsilon & \varepsilon & 1 \\ \delta\varepsilon & \varepsilon & 1 \\ \delta\varepsilon & \varepsilon & 1 \end{bmatrix} 10_j v_d$$

CKM MNS

$$\frac{m_c}{m_t} \simeq \left(1/20\right)^2 \sim \varepsilon^2, \quad \frac{m_u}{m_c} \simeq \left(1/20\right)^2 \sim \delta^2, \quad V_{cb} \cong \left(1/25\right) \sim \varepsilon, \quad V_{ub} \cong \left(1/300\right) \sim \delta\varepsilon,$$

$$\frac{m_s}{m_b} \simeq \left(1/50\right) \sim \varepsilon, \quad \frac{m_d}{m_s} \cong \left(1/20\right) \sim \delta, \quad V_{us} \cong \left(1/5\right) \sim \delta,$$

$$\frac{m_\mu}{m_\tau} \cong \left(1/17\right) \sim \varepsilon, \quad \frac{m_e}{m_\mu} \cong \left(1/205\right) \sim \delta, \quad U_{\mu 3} \simeq (0.7) \sim 1, \quad U_{e3} \leq 0.2 \sim 1,$$

$$\frac{m_2}{m_3} \simeq \left(1/5\right) \sim 1, \quad \frac{m_1}{m_2} \sim ? \sim 1, \quad U_{e2} \simeq (0.55) \sim 1,$$

Is it an accident of the particular example I gave that the 10's transform differently under the gauge group, whereas the $\bar{5}$'s transform the same way?

No: it is typical for economical, anomaly-free, 3-family sets of fermions in SU(N).

The reason is that the 10's must come from tensors of rank $\geq$ 2. These multiplets have a large anomaly compared to the fundamental representation. The most economical choices typically cancel that anomaly with many anti-fundamental multiplets. These contain many $\bar{5}$'s. So most $\bar{5}$'s come from anti-fundamentals.

e.g. most economical SU(7): $3 \cdot [2] + 9[\bar{1}] \rightarrow$ 12 mult., 126 comp.

$[3] + 2 \cdot [2] + 8 \cdot [\bar{1}] \rightarrow$ 11 mult , 133 comp.

$2 \cdot [3] + [2] + 7 \cdot [\bar{1}] \rightarrow$ 10 mult., 140 comp.

$3 \cdot [3] + 6 \cdot [\bar{1}] \rightarrow$ 9 mult, 147 comp.

(others have many more fields, e.g. $4 \cdot [3] + [\bar{2}] + 5[\bar{1}]$ has **196** comp**.**

And most economical SU(8): $[3] + [2] + 9 \cdot [\bar{1}]$

How does the family hierarchy arise?

Quarks and leptons of different families may be in different representations of the grand unified group, or may be different components of the same multiplet.

Therefore: Different terms in the mass matrices may

(a) come from operators of different dimension.
(i) tree-level hierarchy or (ii) radiative hierarchy

(b) contain different Higgs multiplets with VEVs of different magnitude.

(c) contain different components of the same Higgs multiplet, with VEVs of different magnitude.

Consider a SUSY SU(7) example: $10_1 \equiv \psi^{\alpha\beta 7},\ 10_2 \equiv \psi^{\alpha\beta 6},\ 10_3 \equiv \psi^{\alpha\beta},$

$$\bar{5}_1 \equiv \psi_{(1)\alpha},\quad \bar{5}_2 \equiv \psi_{(2)\alpha},\quad \bar{5}_3 \equiv \psi_{(3)\alpha}$$

Higgs: $H^A, \bar{H}_A, H^{AB}, \bar{H}_{AB}, H^{ABC}, \bar{H}_{ABC}$

With a hierarchy of VEVs: $\underbrace{\langle H^{67}\rangle, \langle \bar{H}_{67}\rangle}_{\sim M_P} \gg \underbrace{\langle H^{7}\rangle, \langle \bar{H}_{6}\rangle}_{\sim \varepsilon M_P} \gg \underbrace{\langle H^{6}\rangle, \langle \bar{H}_{7}\rangle}_{\sim \delta\varepsilon M_P}$

$$\langle H^{2}\rangle, \langle \bar{H}_{2}\rangle, \langle H^{267}\rangle, \langle \bar{H}_{267}\rangle \gg \underbrace{\langle H^{27}\rangle, \langle \bar{H}_{26}\rangle}_{\sim \varepsilon M_W} \gg \underbrace{\langle H^{26}\rangle, \langle \bar{H}_{27}\rangle}_{\sim \delta\varepsilon M_W}$$

Consider the up quark mass matrix: M_U

element	operator	order of magnitude
33	$\psi^{\alpha\beta}\psi^{\gamma\delta}H^{267}$	$\sim M_W$
32,23	$\psi^{\alpha\beta}\psi^{\gamma\delta 6}H^{27}$	$\sim \varepsilon M_W$
31,13	$\psi^{\alpha\beta}\psi^{\gamma\delta 7}H^{26}$	$\sim \delta\varepsilon M_W$
22	$\psi^{\alpha\beta 6}\psi^{\gamma\delta\bar{6}}H^{27}H_{\bar{6}} / M_P$	$\sim \varepsilon^2 M_W$
12,21	$\psi^{\alpha\beta 6}\psi^{\gamma\delta\bar{7}}H^{27}H_{\bar{7}} / M_P$	$\sim \delta\varepsilon^2 M_W$
11	$\psi^{\alpha\beta 6}\psi^{\gamma\delta\bar{7}}H^{27}H_{\bar{7}} / M_P$	$\sim \delta^2\varepsilon^2 M_W$

Note:

$$\psi^{ABC}\psi^{DEF}H^{GH} \equiv 0$$

Consider the down quark mass matrix: M_D

element	operator	order of magnitude
3i	$\psi^{\alpha 2}\psi_{(i)\alpha}\bar{H}_2$	$\sim M_W$
2i	$\psi^{\alpha 26}\psi_{(i)\alpha}\bar{H}_{26}$	$\sim \varepsilon M_W$
1i	$\psi^{\alpha 27}\psi_{(i)\alpha}\bar{H}_{27}$	$\sim \delta\varepsilon M_W$

The hierarchy of VEVs ($H^7, \bar{H}_6$ large, $H^6, \bar{H}_7$ small) is easy to achieve:

$$W \supset MH^A\bar{H}_A + H^{AB}\bar{H}_A\bar{H}_B \qquad (M \sim M_P)$$

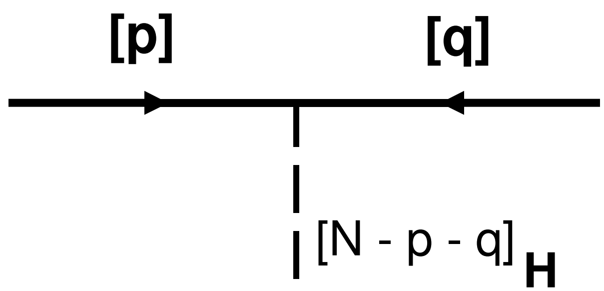
$$\frac{\partial W}{\partial \bar{H}_A} = 0$$

$$\Rightarrow MH^A + H^{AB}\bar{H}_B = 0$$

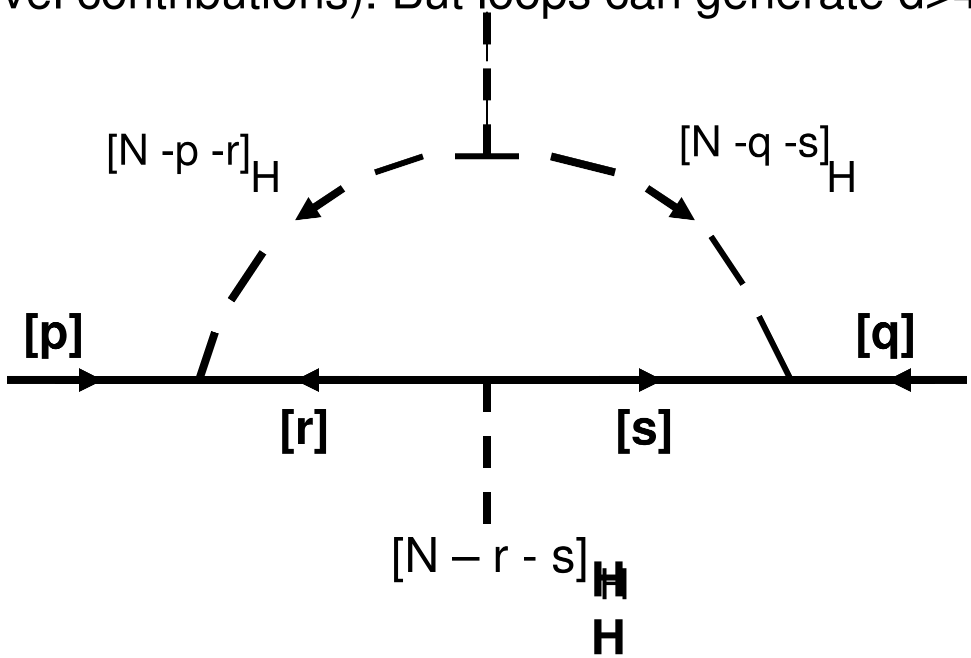
$$\left\langle H^6 \right\rangle \sim \frac{\left\langle H^{67} \right\rangle}{M_P}\left\langle \bar{H}_7 \right\rangle \sim \left\langle \bar{H}_7 \right\rangle, etc$$

Radiative hierarchies are also possible (in non-SUSY GUTS).

The basic idea is very simple (1980):



But if there is no Higgs multiplet [N - p - q], a d=4 operator does not exist (e.g. in the SU(7) example the 12,21,11,22 elements of the up quark mass matrix cannot get tree-level contributions). But loops can generate d>4 operators:



An SU(8) example:

S.M. Barr, Phys. Rev. D78, 075001

hep-ph 0804.1356

- The simplest 3-family set of fermions, and a simple set of Higgs
- No flavor symmetries (the most general couplings)

An SO(10) Radiative Hierarchy Model.

In SO(10), one can also distinguish families by the unified symmetry.
The simplest representations are

$$16 \to 10 + \overline{5} + 1, \qquad 10 \to 5 + \overline{5}, \qquad 45 \to 24 + 10 + \overline{10} + 1$$

So the SU(5)10's can come from different SO(10) multiplets, and similarly for the $\overline{5}$'s

There must be three spinors (16) to get three families, since 10 and 45 are real.
So the **simplest set** of fermions is just three 16's: **no explanation of hierarchy**!

One can add some SO(10) 10's and 45's: in that case, **some families may come from 16's, and others from 10 and 45. That can be achieved without flavor symmetry**. Suppose, for example, a 16 of Higgs that gets a superheavy VEV in the SU(5) singlet direction. Then there are couplings

$$\sum_i c_i (16_i 10) 16_H + M(10 \cdot 10) \to c_i \overline{5}_{(16_i)} 5_{(10)} \left\langle 1_{(16_H)} \right\rangle + \overline{5}_{(10)} 5_{(10)}$$

That means the light $\overline{5}$ will be two linear combinations of $\overline{5}_{(16_i)}$ that are orthogonal to $\Sigma_i c_i \overline{5}_{(16_i)}$ and a linear combination of $\overline{5}_{(16_i)}$ and $\overline{5}_{(10)}$. So one and only one family is partly in the SO(10) 10, the others are purely in 16.

Nevertheless, it is easier to get realistic mass matrices in SO(10) by using a flavor symmetry that distinguishes among the three 16's.

One thereby loses the elegant idea that unified symmetry explains the **whole** flavor structure. But one gains in predictivity.

I will now briefly discuss a **highly predictive non-SUSY SO(10)** model that has a **radiative hierarchy.**

It contains the fermion multiplets $16_{i=1,2,3} + \underline{16 + \overline{16} + 10}$

with renormalizable Yukawa terms (the most general consistent with a U(1) flavor symmetry):

$$L_{YUKAWA} = M_{16}(\overline{16} \cdot 16) + c(10 \cdot 10)54_H$$
$$a(\overline{16} \cdot 16_3)45_H + c_1(10 \cdot 16_1)16_{1H} + c_2(10 \cdot 16_2)16_{2H}$$
$$h_{33}(16_3 \cdot 16_3)10_H + h_2(16 \cdot 16_2)10_H + h_3(10 \cdot 16_3)16'_H$$

The real reps (underlined in red) are integrated out to give **three (tree-level) effective Yukawa operators** that are responsible for all the masses of the heavy two families:

$$O_1 = (16_3 \cdot 16_3)10_H \longrightarrow$$ 33 elements

$$O_2 = (16_2 \cdot 16_3)10_H 45_H \;/\; M_2 \longrightarrow$$ 23,32 elements: antisymmetric, proportional to B-L:

$$O_3 = (c_i 16_i \cdot 16_{iH})(16_3 \cdot 16'_H) \;/\; M_3 \longrightarrow$$ 13, 31, 23, 32 elements: "lopsided", only in down and charged lepton mass matrices

i=1,2

33 elements

Tree Level Mass Matrices:

$$M_U = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \frac{1}{3}\varepsilon \\ 0 & -\frac{1}{3}\varepsilon & 1 \end{pmatrix} m_U, \qquad M_D = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \frac{1}{3}\varepsilon \\ fC_1 & fC_2 - \frac{1}{3}\varepsilon & 1 \end{pmatrix} m_D,$$

$$M_N = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -\varepsilon \\ 0 & \varepsilon & 1 \end{pmatrix} m_U, \qquad M_L = \begin{pmatrix} 0 & 0 & C_1 \\ 0 & 0 & C_2 - \varepsilon \\ 0 & \varepsilon & 1 \end{pmatrix} m_D,$$

This simple structure (only three Yukawa terms!) gives a very good fit to the masses and mixings of the 2nd and 3rd families in terms of only a few parameters. (K.S. Babu and S.M. Barr, PRD 2002)

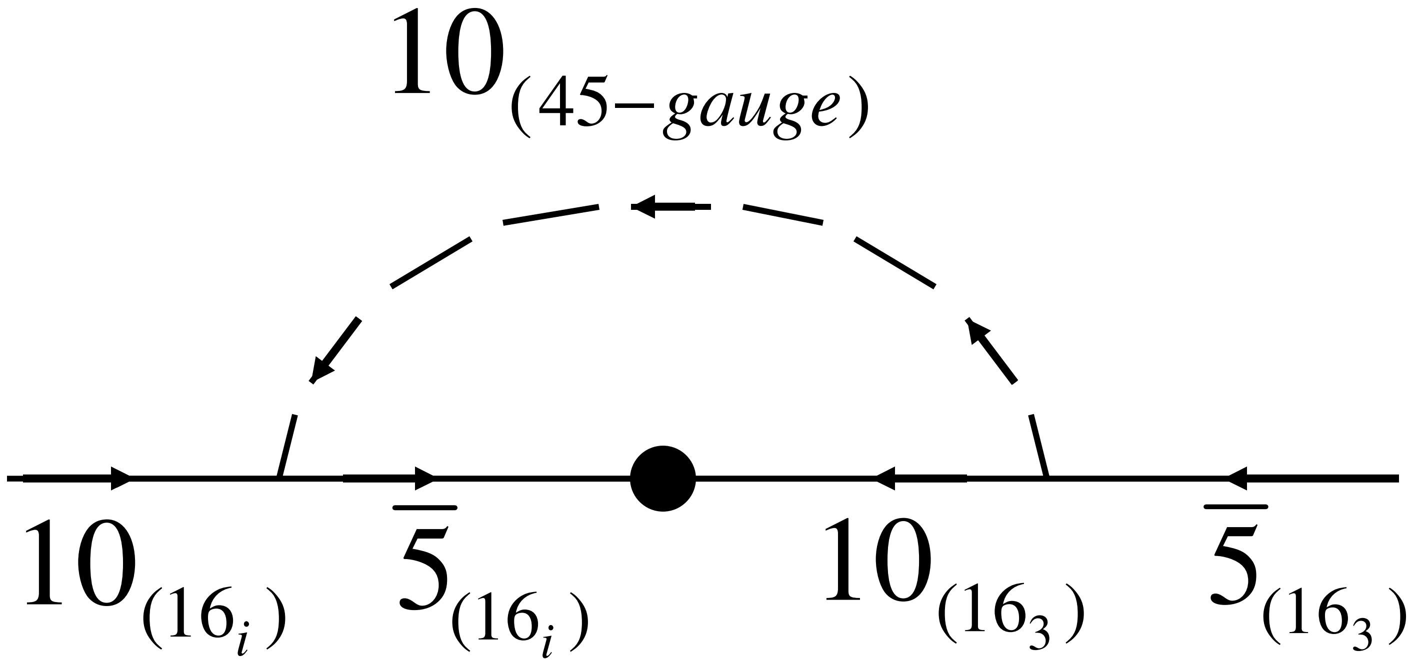
The remarkable thing is that the first family gets mass from loops with almost no new structure having to be added:

A Higgs-loop diagram gives 22 elements

A gauge-boson-loop diagram gives 13 and 31 elements

$$M_U = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \frac{1}{3}\varepsilon \\ 0 & -\frac{1}{3}\varepsilon & 1 \end{pmatrix} m_U, \quad M_D = \begin{pmatrix} 0 & 0 & C_1\delta \\ 0 & \delta_H & \frac{1}{3}\varepsilon + C_2\delta \\ fC_1 & fC_2 - \frac{1}{3}\varepsilon & 1 \end{pmatrix} m_D,$$

$$M_N = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -\varepsilon \\ 0 & \varepsilon & 1 \end{pmatrix} m_U, \quad M_L = \begin{pmatrix} 0 & 0 & C_1 \\ 0 & f_H\delta_H & C_2 - \varepsilon \\ 3C_1\delta & \varepsilon + 3C_2\delta & 1 \end{pmatrix} m_D,$$



$$\Rightarrow \quad \begin{array}{l} \Delta M_{13} \propto M_{31} \\ \Delta M_{23} \propto M_{32} \\ etc. \end{array}$$

	Model	Expt.
m_e	0.00049	0.00049
m_μ	0.1031	0.1032
m_τ	1.756	1.754
m_u	0	0.000571
m_c	0.342	0.278
m_t	87.24	86.93
m_s / m_d	18.68	18.90
m_s	0.0358	0.0254
m_b	1.17	1.186

(in GeV, at M(PS))

	Model	Expt.
V_{us}	0.224	0.224
V_{cb}	0.0456	0.0463
$\lvert V_{ub} \rvert$	0.00368	0.00432
δ_{CKM}	0.887	0.995
$\sin\theta_{sol}$	0.518	0.559
$\sin\theta_{atm}$	0.891	0.707

PARAMETERS:

$\varepsilon = 0.189,\quad 16\pi^2\delta = 2.22,$

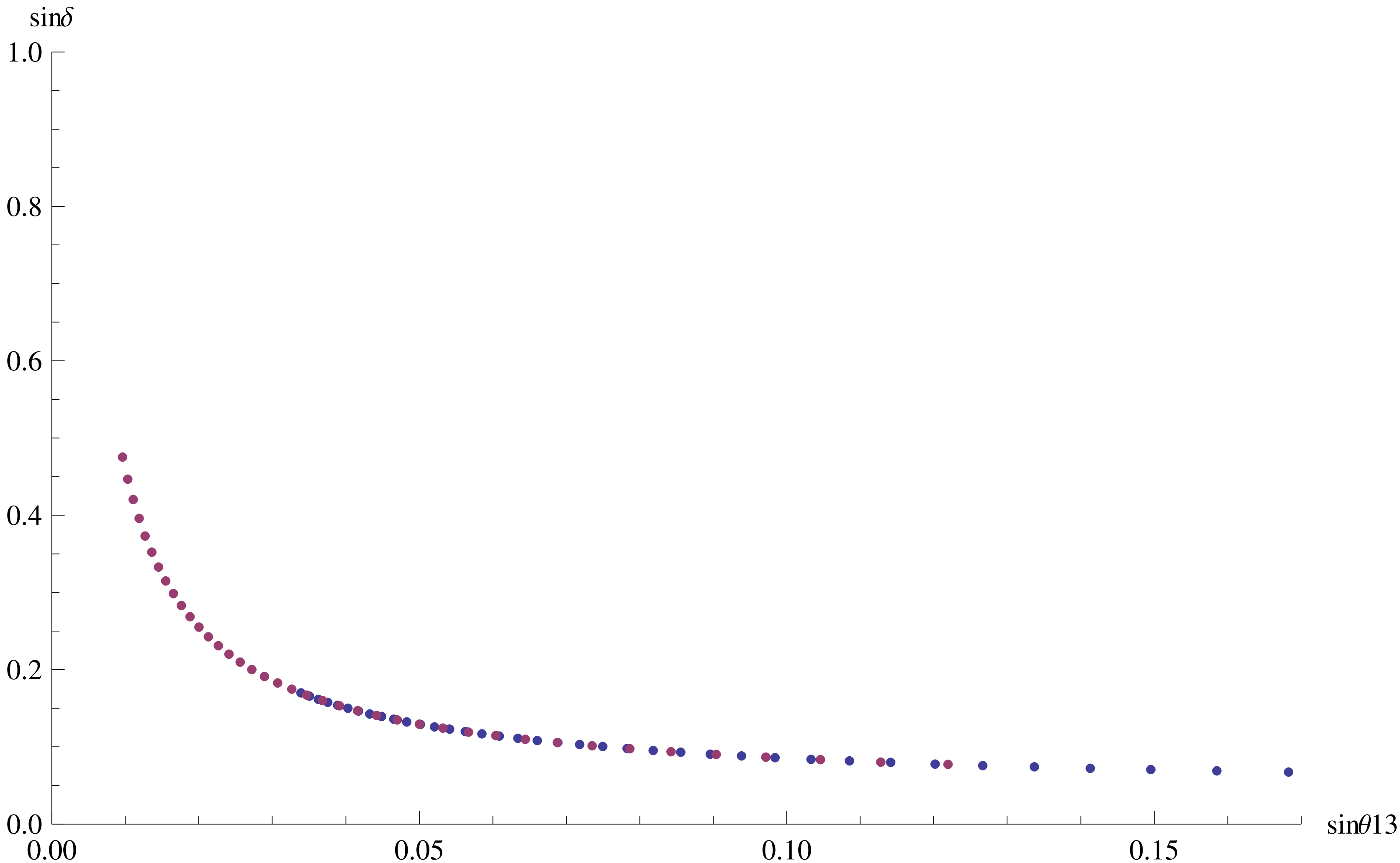
$C_1 = 1.03,\quad 16\pi^2\delta_H = 2.66,$

$C_2 = -1.51,\quad \theta_\varepsilon = 1.52,$

$f = 0.566,\quad \theta_H = 0.514$

$f_H = 0.208,$

$$\chi^2 = 7.2$$


sinδ
1.0
0.8
0.6
0.4
0.2
0.0
0.00
0.05
0.10
0.15
sinθ13