Miami 2008

HD* field theory as the Theory of Everything

based on:

A.S. hep-th/0407231, hep-th/0503213, hep-th/0509022, hep-th/0602201; hep-th/0606139 ;
E.Ivanov + A.S. + B. Zupnik, hep-th/0505082;
E.Ivanov + A.S., hep-th/0510273;
D.Robert + A.S., math-ph/0611023

*) $(HD)_1$ = Higher Dimensional

$(HD)_2$ = Higher Derivative
$(HD)_3$ = Highly Desirable

MOTIVATION

1. Quantum gravity = ?

- problems with unitarity and causality
- they stem from the geometric nature of GR - absence of universal time
- paradoxes also at the classical level - singularities, time machines

String theory ?

- no nonperturbative formulation…

THESAURUS of A.S. in Th. Phys.

1. Second Newton's law
2. Schrödinger equation

Central dogma:

TOE is a field theory in *flat* space-time

- Our Universe = curved 3-brane in flat HD bulk
- gravity is induced

SOAP FILM

$$E = \sigma \mathcal{A} = \sigma \int d^2\xi \sqrt{g}$$

- FSS (Fundamental Theory of Soap) is formulated in *flat* $3D$ space

Theory of the bulk = ?

$$S \sim \frac{1}{\kappa^2} \int d^6x \mathrm{Tr}\{F_{\mu\nu}^2\}$$

involves dimensional coupling and is not renormalizable

We should add more derivatives

$$S \sim \frac{1}{g^2} \int d^6x \mathrm{Tr}\{(\nabla_\mu F_{\mu\nu})^2\}$$

DANGER: the ghosts

- GHOSTS = instability (rather *absence*) of vacuum
- inherent for higher-derivative theories.

Conventional system

$$E = \frac{\dot{q}^2}{2} + V(q)$$

can have a classical and/or quantum bottom

- Let

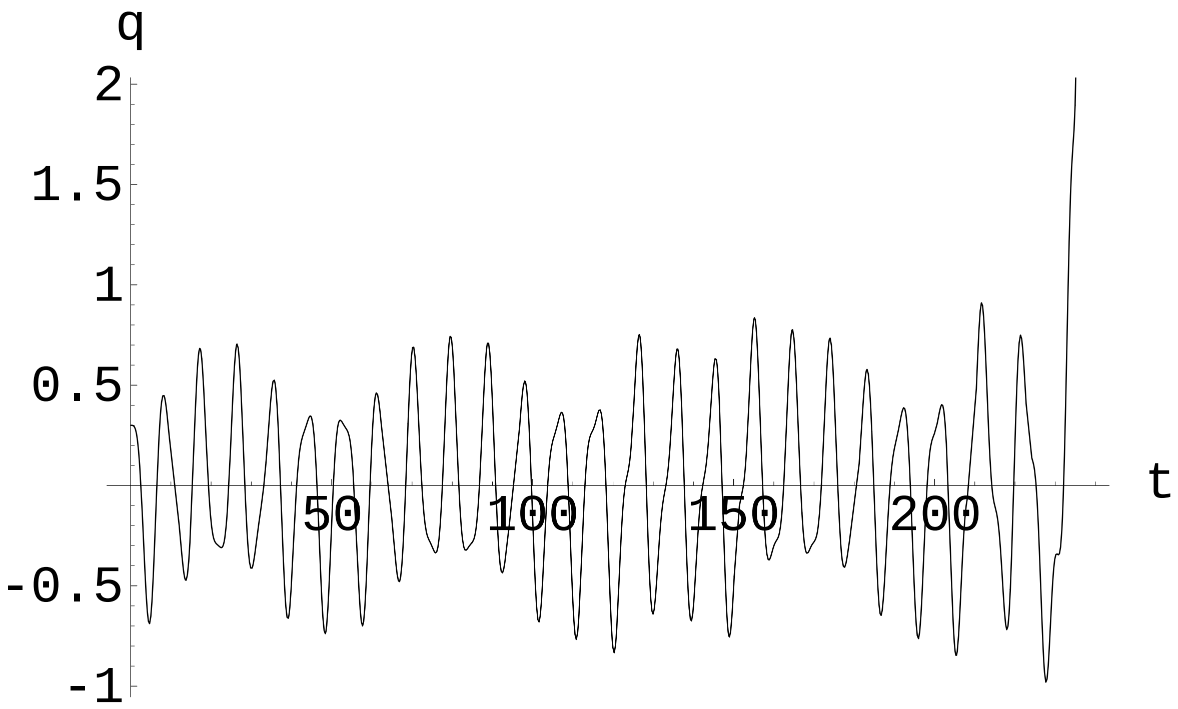
$$L = \frac{1}{2}(\ddot{q} + \Omega^2 q)^2 - \frac{\alpha}{4} q^4 \ .$$

Then

$$E = \ddot{q}(\ddot{q}+\Omega^2 q) - \dot{q}(q^{(3)} + \Omega^2 \dot{q}) - \frac{1}{2}(\ddot{q}+\Omega^2 q)^2 + \frac{\alpha}{4} q^4$$

can be as negative as one wishes.

In this case, classical vacuum $q(0) = \dot{q} = \ddot{q} = \dddot{q} = 0$ is stable with respect to small perturbations.



- The absence of classical bottom does not imply the absence of the quantum bottom.

- Coulomb potential

vs.

- $V(r) = -\gamma/r^2$ with $\gamma < 1/4$.

QUANTUM PAIS-UHLENBECK OSCILLATOR

FREE SYSTEM

$$L = \frac{1}{2}\left[\ddot{q}^2 - (\Omega_1^2 + \Omega_2^2)\dot{q}^2 + \Omega_1^2\Omega_2^2 q^2\right], \qquad \Omega_1 \neq \Omega_2\,.$$

- Ostrogradsky Hamiltonian

$$H = p_q x + \frac{p_x^2}{2} + \frac{(\Omega_1^2 + \Omega_2^2)x^2}{2} - \frac{\Omega_1^2\Omega_2^2 q^2}{2}\,.$$

- Variable change [Mannheim + Davidson, 04]

$$
\begin{aligned}
q &= \frac{a_1}{\sqrt{2\Omega_1(\Omega_1^2-\Omega_2^2)}} + \frac{a_2}{\sqrt{2\Omega_2(\Omega_1^2-\Omega_2^2)}} + \text{h.c.}\ , \\
x &= -\frac{i\Omega_1 a_1}{\sqrt{2\Omega_1(\Omega_1^2-\Omega_2^2)}} + \frac{i\Omega_2 a_2}{\sqrt{2\Omega_2(\Omega_1^2-\Omega_2^2)}} + \text{h.c.}\ , \\
p_x &= -\frac{\Omega_1^2 a_1}{\sqrt{2\Omega_1(\Omega_1^2-\Omega_2^2)}} - \frac{\Omega_2^2 a_2}{\sqrt{2\Omega_2(\Omega_1^2-\Omega_2^2)}} + \text{h.c.}\ , \\
p_q &= \frac{i\Omega_1\Omega_2^2 a_1}{\sqrt{2\Omega_1(\Omega_1^2-\Omega_2^2)}} - \frac{i\Omega_2\Omega_1^2 a_2}{\sqrt{2\Omega_2(\Omega_1^2-\Omega_2^2)}} + \text{h.c.}\ .
\end{aligned}
$$

gives

$$
H = \Omega_1 a_1^* a_1 - \Omega_2 a_2^* a_2\ .
$$

with the spectrum

$$E_{nm} = \left(n + \frac{1}{2}\right)\Omega_1 - \left(m + \frac{1}{2}\right)\Omega_2$$

- positive and negative energies
- dense everywhere if Ω_1/Ω_2 is irrational
- Hamiltonian is Hermitian
- unitary evolution operator exists at all times (no collapse)

The limit $\Omega_1 \to \Omega_2$ is singular

- “Jordanization” of Hamiltonian

$$H \;\to\; \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$$

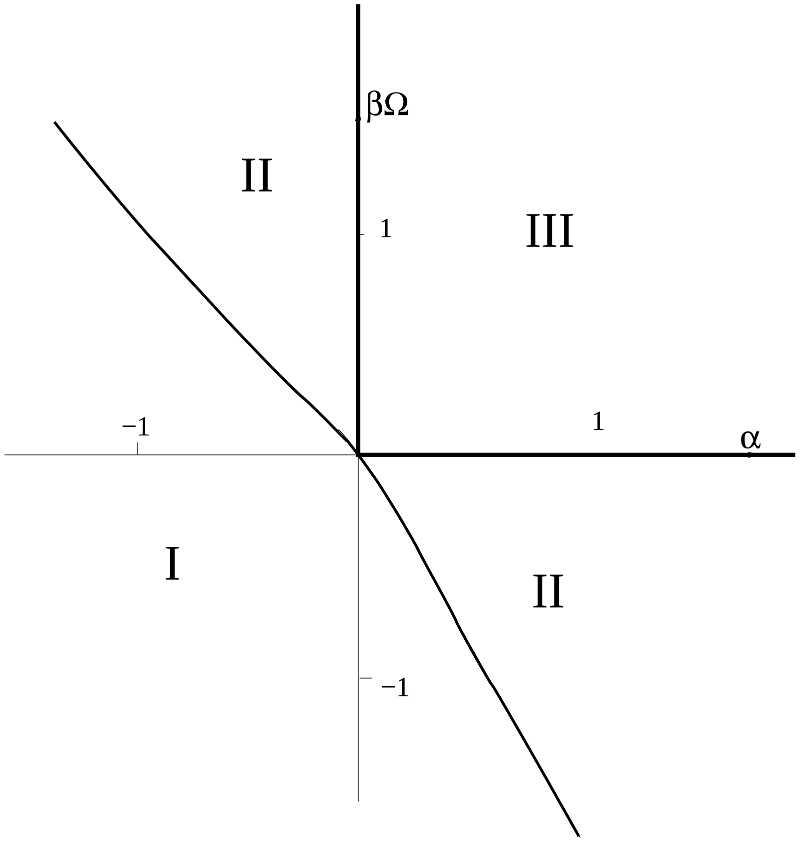
The spectrum: $E_n = n\Omega$ (positive and negative n). Only one eigenstate at each level.

INTERACTING SYSTEM

$$L = \frac{1}{2}(\ddot{q} + \Omega^2 q)^2 - \frac{\alpha}{4} q^4 - \frac{\beta}{2} q^2 \dot{q}^2 \ .$$

- For $\alpha > 0, \beta \geq 0$ or $\alpha \geq 0, \beta > 0$, the spectrum has a bottom !

$$E_0^{\text{var}} \approx -\frac{\Omega^4}{\alpha} \qquad (\beta = 0) \ .$$



Ghost and no-ghost regions. I — perturbative instability. II — falling to the centre. III - Nice Hermitian Hamiltonian endowed with a bottom.

SUPERSYMMETRY vs GHOSTS

- for usual SUSY systems

$$\{\bar{Q}, Q\} = 2H; \qquad Q^2 = \bar{Q}^2 = 0$$

implies $E_n \geq 0$ if $\bar{Q}$ is adjoint to Q.

MODEL

$$L \;=\; \frac{\sigma}{2}\left(\dot{A}_K^2 + \dot{\sigma}^2 + i\psi^{\dagger j}\dot{\psi}_j\right) - \frac{i}{4}\psi^{\dagger j}\gamma_K\psi_j\dot{A}_K + \frac{1}{16\sigma}\left(\psi^{\dagger j}\gamma_0\psi^k\right)\left(\psi_j^{\dagger}\gamma_0\psi_k\right)\;,$$

$K = 1, 2, 3, 4,\;\; j, k = 1, 2;\; \psi^k = \epsilon^{kj}\psi_j.$

- is obtained by dimensional reduction of 5D superconformal theory.
- only $\sigma \geq 0$ is allowed.
- singularity at $\sigma = 0$.

- classical solution: $\sigma = [A+Bt]^{2/3}$; “falls” into the singularity when $B < 0$.

QUANTUM SPECTRUM

- Let $F = 0$.
- Continuous spectrum. The eigenfunctions:

$$\Psi(\sigma, A_M) = g(\sigma) e^{i k_M A_M}$$

with $g(\sigma)$ satisfying

$$-\frac{1}{\sqrt{\sigma}} \left[\frac{\partial^2}{\partial \sigma^2} - k_M^2 \right] \frac{g(\sigma)}{\sqrt{\sigma}} = 2\lambda g(\sigma) \ .$$

- like for the Schrödinger problem with linear potential
- spectral parameter is the *slope* of the potential.

Let $K_M = 0$.

- positive λ solutions

$$g(\sigma) \; = \; \sigma \left[A J_{1/3}\left(\frac{2\sqrt{2\lambda}}{3}\sigma^{3/2}\right) + B J_{-1/3}\left(\frac{2\sqrt{2\lambda}}{3}\sigma^{3/2}\right) \right]$$

- negative λ solutions

$$g(\sigma) \; \sim \; \sigma K_{1/3}\left(\frac{2\sqrt{-2\lambda}}{3}\sigma^{3/2}\right) \qquad (*)$$

- normalizable at $\sigma = 0$. Look benign

Contradiction with supersymmetry ?

- superpartners of (*) are malignant (not normalized)!
- to keep SUSY, (*) should be thrown away (Shifman + A.S. + Vainshtein, 1988)

- complete supermultiplets ($2^4 = 16$ states) have only positive energies

- physically relevant operators should keep the system in reduced Hilbert space (no ghost creation)
- conjecture : this mechanism works also in field theory

SUPERCONFORMAL 5D SYM

$$
\begin{aligned}
g^2\mathcal{L} \;=\; & -\frac{\sigma}{4}F_{\mu\nu}F_{\mu\nu} + \frac{\sigma}{2}(\partial_\alpha\sigma)^2 + \frac{i\sigma}{2}\bar{\psi}^j \not{\partial}\psi_j - \sigma D^{jk}D_{jk} \\
& \frac{i}{8}\bar{\psi}^j\tilde{\sigma}_{\mu\nu}F_{\mu\nu}\psi_j + \frac{1}{24}\epsilon_{\mu\nu\lambda\rho\sigma}A_\mu F_{\nu\lambda}F_{\rho\sigma} + \frac{1}{2}\bar{\psi}^j\psi^k D_{jk}
\end{aligned}
$$

- Vacuum valley for $\sigma \geq 0$.

- If $< \sigma >= m \neq 0$, nonrenormalizable.

$$
\langle\sigma\rangle_{1\ \text{loop}} \;=\; m + \frac{\Lambda^3}{m^2}
$$

- If $< \sigma >= 0$, no quadratic part;

no perturbation theory...

HD SQM model

$$S = \int dt d\bar{\theta} d\theta \left[\frac{i}{2} (\bar{D}X) \frac{d}{dt} (DX) + V(X) \right]$$

- component Hamiltonian

$$H = pP - DV'(x) + \bar{\psi}\bar{\chi} - V''(x)\chi\psi$$

- $N = P^2/2 - V(x)$ is an integral of motion →

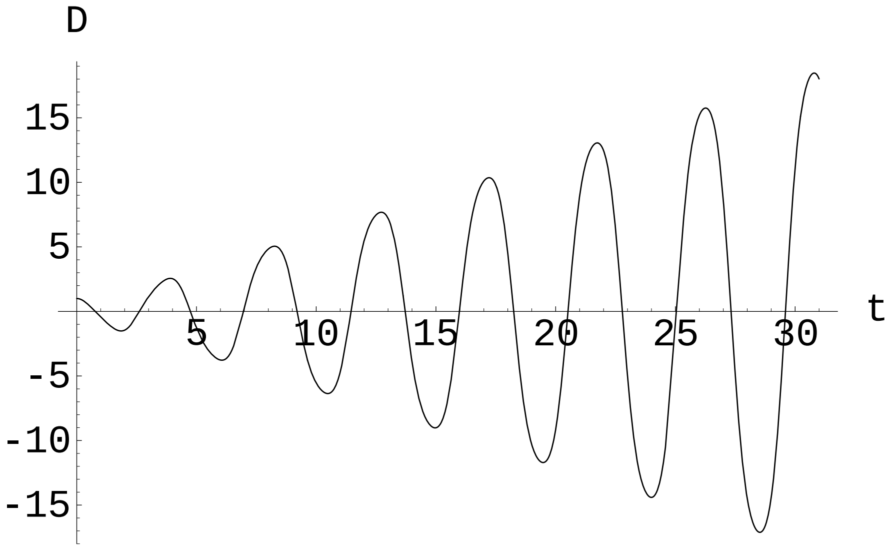
exactly soluble

- for $V(x) = -\lambda X^4/4$

$$D(t) = A \,\mathrm{sn}\left[\Omega t, \sqrt{1/2}\right] \mathrm{dn}\left[\Omega t, \sqrt{1/2}\right] +$$

$$B \left\{ \mathrm{cn}\left[\Omega t, \sqrt{1/2}\right] - \Omega t \,\mathrm{sn}\left[\Omega t, \sqrt{1/2}\right] \mathrm{dn}\left[\Omega t, \sqrt{1/2}\right] \right\} ,$$

with $\Omega^4 = 4\lambda N$.



- exactly soluble, but no toric orbits !
- no collapse → evolution operator is unitary !

Quantum time evolution

(bosonic sector)

- choose mixed representation $\Psi(x, P)$
- Schrödinger equation

$$\frac{\partial \Psi}{\partial t} + P \frac{\partial \Psi}{\partial x} + V'(x) \frac{\partial \Psi}{\partial P} = 0$$

- solution

$$\Psi_t(x, P) = \Psi_0(\Gamma^{-t}(x, P))$$

- full Hamiltonian is not Hermitian
- Nöther supercharges

$$
\begin{aligned}
Q &= \psi[p + iV'(x)] - \bar{\chi}(P - iD) \\
\bar{Q} &= \bar{\psi}(P + iD) - \chi[p - iV'(x)]
\end{aligned}
$$

are not conjugate to each other.

- An extra pair of supercharges

$$
\begin{aligned}
T &= \psi[p - iV'(x)] + \bar{\chi}(P + iD) \\
\bar{T} &= \bar{\psi}(P - iD) + \chi[p + iV'(x)]
\end{aligned}
$$

Full algebra

$$\{\bar{Q}, Q\} = \{\bar{T}, T\} = 2H;$$

$$[\bar{Q}, F] = \bar{Q},\ [Q, F] = -Q, [T, F] = -T, [\bar{T}, F] = \bar{T};$$

$$[Q, N] = [T, N] = \frac{Q - T}{2}, -[\bar{Q}, N] = [\bar{T}, N] = \frac{\bar{Q} + \bar{T}}{2}$$

with $F = \psi\bar{\psi} - \chi\bar{\chi}$ = fermion charge.

Spectrum (for $V(X) = -\omega^2 X^2/2 - \lambda X^4/4$) :
$(-\infty, -\omega] \cup \{0\} \cup [\omega, \infty)$

- No contradiction with supersymmetry as $\bar{Q}$ is not conjugate to Q :
- Supersymmetric quartets

- in spite of non-Hermiticity of Hamiltonian, the spectrum is real.
- belongs to the class of quasi-Hermitian, alias crypto-Hermitian systems. (ask Carl Bender)

- Conventional $6D$ superspace:

$$z = (x^M, \theta^a_i) \quad (M = 0, \ldots, 5, a = 1, \ldots, 4, i = 1, 2)$$

with pseudoreality constraint

$$\theta^a_i = -\epsilon_{ij} C^{ab} (\theta^b_j)^*$$

- complex conjugated spinor $(0, 1)$ is not $(1, 0)$.

Specific for $SO(5, 1)$.

$6D$ Harmonic superspace

- Introduce coset $CP^1 = SU(2)/U(1)$ with coordinates $u^{\pm i}$ $(u^{-i} = (u^{+i})^*, u^{+i}u_i^- = 1)$.
- Introduce $\theta^{\pm a} = u_k^{\pm}\theta^{ak}$
- Grassmann-analytic superfields depend only on

$$(\zeta, u) = (x_A^M, \theta^{+a}, u^{\pm i}), \qquad x_A^M = x^M + i\theta^{+a}\gamma_{ab}^M\theta^{-b}$$

- only 16 components in θ expansion. But infinite number of components in harmonic expansion over $u^{\pm}$.
- Basic gauge superfield $V^{++}(\zeta, u)$.
- gauge freedom $\delta_\Lambda = D^{++}\Lambda - [V^{++}, \Lambda]$.
- Only a finite number of physical components are left.

SUPERCONFORMAL 6D SYM

$$
\begin{aligned}
S = -\frac{1}{g^2}\int d^6x\,\mathrm{Tr}\Big\{ & \left(\nabla^M F_{ML}\right)^2 + i\psi^j\gamma^M\nabla_M(\nabla)^2\psi_j + \frac{1}{2}\left(\nabla_M\mathcal{D}_{jk}\right)^2 \\
& + \mathcal{D}_{lk}\mathcal{D}^{kj}\mathcal{D}_j{}^{l} - 2i\mathcal{D}_{jk}\left(\psi^j\gamma^M\nabla_M\psi^k - \nabla_M\psi^j\gamma^M\psi^k\right) + \left(\psi^j\gamma_M\psi_j\right)^2 \\
& + \frac{1}{2}\nabla_M\psi^i\gamma^M\sigma^{NS}[F_{NS},\psi_j] - 2\nabla^M F_{MN}\,\psi^j\gamma^N\psi_j\Big\}\ .
\end{aligned}
$$

- degrees of freedom: $2\times 5-1=9$ for A_M, 3 for $\mathcal{D}_{jk}$ and $4\times 3=12$ for ψ^{ja} (and multiply by N^2-1).

- at the classical level, enjoys $\mathcal{N}=1$ 6D superconformal invariance.

- propagators:

$$
\begin{aligned}
\langle A_M^A A_N^B \rangle &= -\frac{i\eta_{MN}\delta^{AB}}{p^4}\,, \\
\langle \psi^{jA}\psi^{kB} \rangle &= -\frac{i\epsilon^{jk}\delta^{AB}p_N\tilde{\gamma}^N}{p^4}\,, \\
\langle \mathcal{D}_{ik}^A \mathcal{D}_{jl}^B \rangle &= -\frac{i\delta^{AB}}{p^2}\left(\epsilon_{ij}\epsilon_{kl}+\epsilon_{il}\epsilon_{kj}\right),
\end{aligned}
$$

conformal anomaly

- effective lagrangian

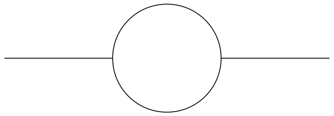
$$\begin{aligned}\mathcal{L}_{\mathcal{D}}^{\mathrm{eff}} &= -\frac{1}{2}\mathrm{Tr}\left\{(\partial_M \mathcal{D}_{jk})^2\right\}\left(1+\frac{g_0^2 c_V}{48\pi^3}\ln\frac{\Lambda}{\mu}\right) \\ &\quad -g\mathrm{Tr}\left\{\mathcal{D}_{lk}\mathcal{D}^{kj}\mathcal{D}_j^l\right\}\left(1-\frac{7g_0^2 c_V}{96\pi^3}\ln\frac{\Lambda}{\mu}\right).\end{aligned}$$

- effective charge

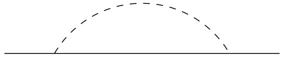
$$g(\mu) = g_0\left(1-\frac{5g_0^2 c_V}{48\pi^3}\ln\frac{\Lambda}{\mu}\right)$$

(Landau zero)

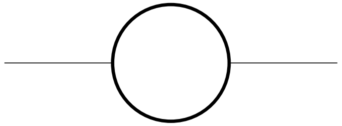
- relevant graphs:



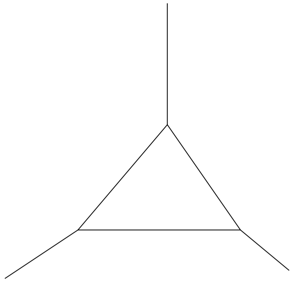
a)



b)



c)



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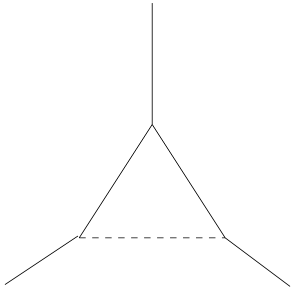

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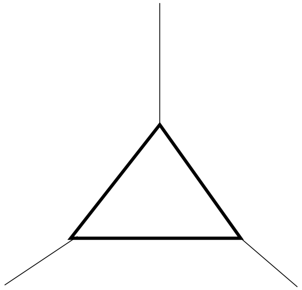

c)

dashed - A_M, solid - $\mathcal{D}$, thick solid - fermions.

gauge chiral anomaly

$$\nabla_M J^a_M = -\frac{1}{3 \cdot 128\pi^3} \epsilon_{ABCDEF} \mathrm{Tr}\{T^a \hat{F}_{AB} \hat{F}_{CD} \hat{F}_{EF}\} \ ,$$

$(T^a)_{bc} = -if^{abc}, \hat{F}_{\alpha\beta} = F^a_{\alpha\beta} T^a.$

- chiral asymmetry of the fermion action
- technical reason: $\mathrm{Tr}\{T^{(a}T^bT^cT^{d)}\} \neq 0.$

(cf. $\mathrm{Tr}\{T^{(a}T^bT^{c)}\} = 0$ for 4D)

- can be compensated by adding an adjoint hypermultiplet.

A) Conventional hypermultiplet

$$\mathcal{L} \sim \int d\zeta du \, q^+ \nabla^{++} q^+$$

- involves the fermion of opposite compared to "gluino" chirality, which cancels anomaly.
- unnatural dimensions $[\phi] = 2, [\psi] = 5/2$. $L_{\text{free}} \sim (\partial\phi)^2 + \psi \partial\!\!\!/ \psi$.
- contibutes to conformal anomaly with the same sign as pure gauge.

B) Higher derivatives.

- conventional dimensions $[\phi] = 1, [\psi] = 3/2$.
- conformal invariance is lost already at the classical level.
- Infinite number of dynamic fields.

$$\mathcal{L}_{\text{ferm kin}} \sim \int du \, \mathrm{Tr}\{ i\chi_a^k (\tilde{\gamma}^\mu)^{ab} \partial_\mu \Box \chi_{kb} - 2\chi_a^k \, u^{+i} \frac{\partial}{\partial u^{-i}} \Box \lambda_k^a \} \, ,$$

with

$$\begin{aligned} \chi(u,x) &= \chi(x) + \chi^{(ij)}(x) u_i^+ u_j^- + \chi^{(ijkl)} u_i^+ u_j^+ u_k^- u_k^- + \ldots \\ \lambda(u,x) &= \lambda^{(ij)}(x) u_i^- u_j^- + \lambda^{(ijkl)} u_i^- u_j^- u_k^- u_l^+ + \ldots \end{aligned}$$

- the same contribution to the anomaly as the conventional hypermultiplet.
- infinite contribution to the beta function ??

Mathematical fact:

- superconformal algebra exists only at $D \leq 6$.
- two variants at $D = 6$:

minimal ($\mathcal{N} = 1$) and extended ($\mathcal{N} = 2$)

OUR BET:

TOE = maximally superconformal theory

- dual to $AdS_7 \otimes S^4$.
- no explicit field theory realization is known