The Delta-Nucleon Electromagnetic Interaction and Wigner Rotations

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Abstract

We explore some aspects of an infinite momentum frame nonperturbative, relativistic sum rule approach to the study of the $\Delta N \gamma$ electromagnetic interaction where we find that Wigner rotations play an important role and that it may be plausible that neglect or improper usage of Wigner rotations could have significant effects on the experimental and theoretical determination of interaction observables.

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Introduction—Basic Theoretical and Experimental Problems

- The study of the $\Delta N\gamma$ transition form factors $G_M^*(q^2)$ and $G_E^*(q^2)$ and $G_C^*(q^2)$ associated with the $\Delta(1232)$ nucleon resonance isobar has engendered much experimental and theoretical research for several decades.

- On the other hand—as the most studied nucleon resonance—the $\Delta(1232)$ has proved to be very difficult in the determination of its physical properties vis-á-vis its relation to the nucleon[1–16].

- Although similar to the nucleon in valence quark content, it has spin and isospin of 3/2 and its interaction with other particles via form factors is much more complex than that of the nucleon.

- The $\Delta(1232)$ is unstable, so measurement of physical observables and theoretical modeling is difficult.

- Of special interest is the $\Delta(1232)$–nucleon 4-vector electromagnetic current matrix element ($\langle N|\, j^\mu \,|\Delta\rangle$) associated with the process $\Delta \longleftrightarrow N+\gamma^*$ described covariantly by the form factors $G^*_M(q^2)$, $G^*_E(q^2)$, and $G^*_C(q^2)$, where q^2 is the photon 4-momentum transfer squared. This matrix element and associated form factors is important in pion photoproduction and electroproduction (*i.e.* $\pi N \rightarrow \Delta \rightarrow \pi N\gamma$ or πN and $\gamma N \rightarrow \Delta \rightarrow \pi N$ or $\pi N\gamma$).

- In a perfect world with unbroken $SU_F(N)$ flavor symmetry, one expects that $G^*_E(q^2) = G^*_C(q^2) = 0$ and that $G^*_M(q^2)$ would exhibit the same q^2 behavior as does the Sachs nucleon form factor G_M thus giving rise to *pure* magnetic dipole $\Delta N\gamma$ transitions. Instead, one unexpectedly finds that, experimentally, G^*_M decreases faster as a function of $Q^2 \equiv -q^2$ than does G_M, the ratio $-G^*_E/G^*_M \neq 0$, and that G^*_M possesses a complicated behavior as a function of Q^2.

- General field-theoretic equal time commutation relations (ET(CR)s) exist from which sum rules may be generated which relate vector and axial-vector currents among each other[17]. One can produce sum rules which relate $\langle N|\, j^\mu \,|\Delta\rangle$ to nucleon matrix elements involving j^μ (*i.e.* $\langle N|\, j^\mu \,|N\rangle$) *and* the axial-vector current A^μ_π (*i.e.* $\langle N|\, A^\mu_\pi \,|N\rangle$).

- The possible existence of the nucleon weak axial-vector second class current (SCC)[1] form factor $g_T(q^2)$ present in the matrix element $\langle p| A^{\mu}_{\pi^+} |n\rangle$ has also engendered some experimental and theoretical research for several decades and it is clear that extant SCC effects may play some role in the determination of a correct description of the matrix element $\langle N| j^{\mu} |\Delta\rangle$, much more research needs to be done in this area.

- At present—utilizing the Standard Model (SM) theoretical framework which does not explicitly contain SCC currents—SCC effects can not be ruled out definitively in many nuclear β-decay processes, neutrino (antineutrino)-nucleon scattering reaction processes (including those involving τ lepton production and τ decays), semi-leptonic hyperon β-decay processes, neutrino oscillation phenomena, and the astrophysical implications of such oscillations.

- It is clearly possible that the inclusion of SCC effects normally neglected in most experimental and theoretical studies in general and in pion photoproduction and electroproduction processes in particular, may aid in our understanding of the basic underlying connecting physics principles responsible for such processes.

- Another subject of high interest is associated with pion electroproduction processes where the relationship between the $\Delta \to N + \gamma$ transition form factors and nucleon form factors—pertaining to leading-order perturbative QCD (PQCD) predictions for the ratios $R_{EM} \equiv E_{1^+}/M_{1^+}$, $R_{SM} \equiv S_{1^+}/M_{1^+}$, and the photon helicity amplitudes $A_{1/2}$ and $A_{3/2}$ at high momentum transfer—bears special scrutiny. Indeed, PQCD predicts $R_{EM}(q^2) \to 1$ and $R_{SM}(q^2) \to$ *constant* but leaves open the question as to the region of q^2 where this behavior is manifested.

- Experimentally, however, the R_{EM} appears to be non-zero, small in magnitude, and of the order of a few percent. Most analyses predict the R_{EM} to be small and negative at small momentum transfer. The quantity $R_{SM} \equiv S_{1^+}/M_{1^+} \propto G^*_C/G^*_M$ is also predicted to be non-zero, negative, but larger in magnitude than R_{EM}. As pointed out by us [18], R_{SM} is *equally important* in investigations of Δ electroproduction processes.

- A subject which we think deserves more attention in its application to the study of $\Delta N\gamma$ interactions is the Wigner Rotation[19]. The reason is very simple: Most often matrix elements describing observables of interest are best constructed using the helicity basis or in terms of helicity amplitudes, given the fact that helicity is invariant under Lorentz boosts along a particle's direction of motion and under spatial rotations as well. Thus, a sequence of arbitrary boosts applied to a massive particle with spin initially at rest which results in the particle being brought back to rest will leave the particle rotated by a angle which is called the Wigner angle or rotation. As a consequence transformations of matrix elements involving particles from one frame to another will bring into play Wigner rotations (specific to each particle described in the matrix element) which implies helicity mixing. As many observables of interest are best written in terms of specific helicity form factors (*transverse one-half, longitudinal, etc.*, it is crucial that Wigner rotation effects are taken into account properly. We note that while the calculation of a Wigner rotation is straight-forward and a kinematic effect of geometric origin, in practice, it can be laborious and difficult, particularly when multi-particle processes are involved.

Introduction—Basic Theoretical Concepts and Methodology

Research Outline and Overview

- Use algebraic methods which incorporate Quantum Chromodynamic (QCD) equal time commutation relations (ETCRs)and a nonperturbative, relativistic sum rule approach in the infinite momentum frame where $G_M^*(q^2)$ and $G_E^*(q^2)$ are calculated in terms of $g_T(q^2)$ and the well-known nucleon isovector Sachs form factor G_M^V as input with no additional model parameters[20].

- These methods generate algebraic sum rules consisting of covariant matrix elements (and also *gauge invariant* for matrix elements involving the electromagnetic current) which can be used to study—nonperturbatively—a wide variety of strong and electroweak processes important for the advancement of our knowledge of nuclear and elementary particle physics.

- Contrary to popular belief in some physics circles, the use of current-algebra techniques to solve difficult *nonperturbative* problems in nuclear and elementary particle physics continues to be timely, worthwhile, and extremely effective.

- Why? General relativistic quantum field theory as expressed by people such as Lehmann, Symanzik, Zimmermann, and Wightman (and many others) is very much essential to our understanding of physics at its deepest most fundamental level. Indeed, one hopes to construct models which impose minimal physical restrictions, remain mathematically tractable, but yet still allow one to make predictions amenable to experimental verification. These considerations, taken together, result in the concepts of ETCRs and the S matrix, which when examined in model Hilbert spaces ultimately yield relations involving matrix elements of *currents*.

The matrix elements of the weak axial-vector current between the proton and the neutron [Eq. (1)], the virtual process $p \to p + \gamma$ [Eqs. (2) and (3)], and the $\Delta N\gamma$ vertex [Eqs. (4) and (5)] are given by:

$$
\begin{aligned}
&(2\pi)^3 \sqrt{\frac{E_{\tilde{p}^*} E_{\tilde{p}}}{m m_n}} \langle p(\tilde{p}, \lambda) | \, A^{\mu}_{\pi^+}(0) \, | n(\tilde{p}^*, \lambda^*) \rangle \\
&= \bar{u}_p(\tilde{p}, \lambda) \left[\{ g_A(\tilde{q}^2) \gamma^\mu + i g_T(\tilde{q}^2) \sigma^{\mu\nu} \tilde{q}_\nu + g_p(\tilde{q}^2) \tilde{q}^\mu \} \gamma_5 \right] u_n(\tilde{p}^*, \lambda^*) \, ; \qquad (1)
\end{aligned}
$$

$$
(2\pi)^3 \sqrt{\frac{E_{\tilde{p}^*} E_{\tilde{p}}}{m^2}} \langle p(\tilde{p}, \lambda) | \, j_\mu(0) \, | p(\tilde{p}^*, \lambda^*) \rangle = e \, \bar{u}_p(\tilde{p}, \lambda) \, [\Gamma_\mu] \, u_p(\tilde{p}^*, \lambda^*) \, , \qquad (2)
$$

$$
\Gamma_\mu \equiv [1 - \tilde{q}^2/(4m^2)]^{-1} [(i/(4m^2)) G_M(\tilde{q}^2) \epsilon_\mu \left(\tilde{P} \tilde{q} \gamma \right) \gamma_5 + (1/(2m)) G_E(\tilde{q}^2) \tilde{P}_\mu] \, ; \qquad (3)
$$

$$
\langle p(p, \lambda) | \, j_\mu(0) \, | \Delta^+(p^*, \lambda^*) \rangle = \frac{e}{(2\pi)^3} \sqrt{\frac{m m^*}{E_p E_\Delta}} \bar{u}_p(p, \lambda) \, [\Gamma_{\mu\beta}] \, u^\beta_\Delta(p^*, \lambda^*) \, , \qquad (4)
$$

$$
\begin{aligned}
\Gamma_{\mu\beta} \equiv & -i \frac{3(m^* + m)}{2m} (G^*_M - 3G^*_E)(Q^+ Q^-)^{-1} m^* q_\beta \epsilon_\mu(qp\gamma) \\
& - \frac{3(m^* + m)}{2m} (G^*_M + G^*_E)(Q^+ Q^-)^{-1} [2\epsilon_{\beta\sigma}(p^* p) \epsilon_\mu^{\ \sigma}(p^* p) \gamma_5 - i m^* q_\beta \epsilon_\mu(qp\gamma)] \\
& + \frac{3(m^* + m)}{m} G^*_C (Q^+ Q^-)^{-1} q_\beta [p \cdot q q_\mu - q^2 p_\mu] \gamma_5 \, , \qquad (5)
\end{aligned}
$$

- Here $e^2 = 4\pi\alpha$, $\alpha \equiv$ fine-structure constant, $q \equiv p^* - p$, $p^* = (p^{*0}, \vec{t}\,)$ and $p = (p^0, \vec{s}\,)$ are the four-momenta of the Δ^+ and nucleon respectively, $m^* = \Delta^+$ mass, $m =$ proton mass $\approx$ neutron mass $= m_n$, $\tilde{q} = \tilde{p}^* - \tilde{p}$, $\tilde{p}^* = (\tilde{p}^{*0}, \vec{t}\,)$ and $p = \tilde{p} = (\tilde{p}^0, \vec{s}\,)$, $p^{*2} = m^{*2}$, $\tilde{p}^{*^2} = m^2$, $Q^\pm \equiv (m^* \pm m)^2 - q^2$, $\tilde{q}^2_\pm = [(m^{*2} + m^2)q^2 + (m^{*2} - m^2)(\pm\sqrt{Q^+Q^-} - (m^{*2} - m^2))]/(2m^{*2})$, λ and λ^* are particle helicities, $\tilde{Q}^+_\pm \equiv 4m^2 - \tilde{q}^2_\pm$, and $\tilde{Q}^-_\pm \equiv -\tilde{q}^2_\pm$.

- g_A, g_T, and g_P are the nucleon axial-vector, induced pseudotensor, and induced pseudoscalar form factors respectively. The kinematic singularity-free $\Delta N\gamma$ transition form factors are $G^*_M(q^2)$, $G^*_E(q^2)$, and $G^*_C(q^2)$ and the first, second, and third terms in Eq.(5) induce transverse $\frac{1}{2}$, transverse $\frac{3}{2}$, and longitudinal helicity transitions, respectively, in the rest frame of Δ.

- Under G-parity transformations, g_A and g_P are represented by first-class currents (FCC) while g_T is represented by SCC. In the SM, a non-zero g_T can only arise due to quark mass and charge differences and thus the ratio g_T/g_A is thought to be very small or identically zero, even though SCC pseudotensor and SCC scalar form factor data independent of CVC and PCAC assumptions is not yet precise.

Assuming no SCC effects—see Ref.[21]—the $\Delta N\gamma$ transition form factors $G^*_M(q^2)$ and $G^*_E(q^2)$ and $G^*_C(q^2)$ can be calculated in terms of the well-known nucleon isovector Sachs form factor parametrized by $G^V_M(\tilde{q}^2) = \frac{1}{2}(\mu_p - \mu_n)G_{\rm dipole}(\tilde{q}^2)$, with $G_{\rm dipole}(\tilde{q}^2) \equiv [1 - \tilde{q}^2/0.71\ GeV^2/c^2]^{-2}$, where μ_p and μ_n are the proton and neutron magnetic moments respectively [Eqs. (8) and (7) are helicity nonflip and helicity flip sum rules respectively and $j^\mu \equiv j^\mu_3 + j^\mu_S$, where $j^\mu_3 \equiv$ isovector part of j^μ and j^μ_S is isoscalar] :

$$< p(\vec{s}, +1/2\,)|j^\mu(0)|\,\Delta^+(\vec{t}, +1/2\,) > = \frac{5\sqrt{2}}{4}\left\{- \frac{\left\langle p(\vec{s}, +1/2\,)\right|\, A^\mu_{\pi^+}(0)\left| n(\vec{t}, +1/2\,)\right\rangle}{2g_A(0)} + \left\langle p(\vec{s}, +1/2\,)\,|j^\mu_3(0)|\,p(\vec{t}, +1/2\,)\right\rangle\right\}, \qquad (6)$$

$$< p(\vec{s}, -1/2\,)\,|j^\mu(0)|\,\Delta^+(\vec{t}, +1/2\,) >= \frac{5}{8}\sqrt{2}\,\langle p(\vec{s}, -1/2\,)\,|\, j^\mu_3(0)\left| p(\vec{t}, +1/2\,)\right\rangle . \qquad (7)$$

- In obtaining Eqs. (8) and (7), one uses the ETCR $[\, j_3^\mu(0), A_{\pi^+}] = A^\mu_{\pi^+}(0)$ and infinite momentum frame $[\, |\vec{s}| \to \infty, |\vec{t}| \to \infty]$ level realization *which does not require saturation or ad hoc truncation of intermediate states*.

- We do this by inserting the ETCR between (baryon) states $\left\langle \alpha', \lambda', \vec{s}, D', {L'}^{P'}_{N'} \right|$ and $\left| \alpha, \lambda, \vec{t}, D, L^P_N \right\rangle$, where α, α' denote physical flavor indices (*i.e.* $n, p, \Delta^+, ...$), $\lambda', \lambda =$ *particle helicity, hadronic levels are labeled by* $L^P_N, {L'}^{P'}_{N'} = 0^+_0, 0^+_2, 1^-_1, ...,$ *whereas flavor multiplet (*i.e. singlet, octet, nonet, decuplet,...) and J^P are labeled by D, D' to which a particle belongs.

- In the present case, we choose the ground state octet and decuplet baryons as external states, so that $D = D'$ and $L^P_N = {L'}^{P'}_{N'} = 0^+_0$. One then inserts a complete set($\Sigma_{D'',L''}$) of internal states grouped by appropriate labels D'' and ${L''}^P_N$. *To each of these groupings is associated a fractional level parameter* $f(D'', {L''}^P_N, \lambda', \lambda)$ *which does not depend on* α' *or* α *and is required to be invariant under flavor group rotations*.

- Effectively, one has replaced a single CR sum rule with an infinite number of grouped internal states with an *equivalent* infinite number of CRs with a single grouping of internal states where the right hand side of each CR is now multiplied by the parameter $f(D'', {L''}^P_N, \lambda', \lambda)$.

- Eqs. (8) and (7) were obtained by first using the ground state baryons ($D = \overbrace{octet, decuplet}$, $L^P_N = 0^+_0$) as external states and choosing the internal grouping to be the same as the external states; second, selecting only those sum rules with the same associated parameter $f(\overbrace{octet, decuplet}, 0^+_0, \lambda', \lambda)$ (resulting in a set of nonlinear matrix element equations), and finally by eliminating the parameter $f(\overbrace{octet, decuplet}, 0^+_0, \lambda', \lambda)$ itself. [20].

It is important to note (and is the major point in this talk) that Eq. (5) describes the $\Delta N\gamma$ electromagnetic interaction in the Δ rest frame (nucleon and Δ helicities and momenta specified) and when used must be suitably transformed for whatever frame of reference that is required. This transformation can be done and brings into play the Wigner angles for the Δ and the N. It may also be advantageous for one make a transformation in such a frame as to make calculation easier. *This transformation and its accompanying Wigner angles may also be responsible at some level for some theoretical/experimental difficulties in calculating/determining observables.*

Let's briefly illustrate the helicity representation transformation law for matrix elements:

Let G be an arbitrary proper Lorentz transformation and $H(\overrightarrow{gp})$ define the transformation for the helicity state $|\vec{p}, s, \lambda >= |p\theta\phi s\lambda >= R(\theta,\phi)|ps\lambda >= H(\overrightarrow{gp})|s\lambda >$

Then we have:

$G|\vec{p}, s, \lambda >= \Sigma_{\lambda'} |\overrightarrow{gp} s\lambda' >< (\vec{s}\lambda'|R_g|s\lambda >$

$R_g = H^{-1}(\overrightarrow{gp})GH(\vec{p})$ so that

$G|\vec{p}, s, \lambda >= \Sigma_{\lambda'} |\overrightarrow{gp} s\lambda' > D^{(s)}_{\lambda',\lambda}(R_g)$.

R_g and $R_{g'}$ are Wigner rotations (defined individually) for the initial and final states above.

Now, if A is an operator which transforms like $GAG^{-1} = A_G$ then one obtains:

$$< \vec{p'} s'\mu'|A|\vec{p}s\mu >= \sum_{\lambda\lambda'} D^{(s')}_{\lambda',\mu'}(R_{g'}) < \vec{p'} s'\mu'|A_G|\vec{p}s\mu' > D^{(s)}_{\lambda,\mu}(R_g) \qquad (8)$$

A very useful relation!

Example

When $G_\Delta : \vec{p} \to (m_\Delta, 0, 0, 0)$ and $A_G = j^{1-i2}_{EM}$, $A = A_G = j^{1-i2}_{EM}$, $\mu' = \frac{1}{2} = \mu$, we find that $D^{(s')}_{-1/2,1/2}(R_{g'}) = \sin(\frac{mp_z p'_x}{m_\Delta |\vec{p'}|^2})$ so that the nucleon transverse momentum can become very important.

Conclusions

WATCH OUT FOR WIGNER ROTATIONS!!

I. BIBLIOGRAPHY

[1] S. Weinberg, Phys. Rev. **112**, 1375 (1958).

[2] B. Holstein, Phys. Rev. **C4**, 764 (1971).

[3] K. Kubodera, J. Delorme, and M. Rho, , Phys. Rev. Lett. **38**, 321 (1977).

[4] E. H. Monsay, Phys. Rev. **D16**, 609 (1977).

[5] H. F. Jones and M. Scadron, Ann. Phys. **81**, 1 (1973); R. C. E. Devenish, R. S. Eisenschitz, J. G. Korner, Phys. Rev. D **14**, 3063 (1977); L. Durand, P. DeCelles, and R. Marr, Phys. Rev. **126**, 1882 (1962).

[6] L. A. Ahrens, *et al.*, Phys. Lett. **202B**, 284 (1988).

[7] H. Pietschmann and H. Rupertsberger, Phys. Rev. **D40**, 3115 (1989).

[8] P. Stoler, Phys. Rep. **226**, 103 (1993).

[9] H. Shiomi, Nucl. Phys. **A603**, 281 (1996).

[10] T. Minamisono *et al.*, Phys. Rev. Lett. **80**, 4132 (1998).

[11] D. H. Wilkinson, Nucl. Instrum. Meth. **A455**,656 (2000).

[12] K. Minamisono *et al.*, Nucl. Phys. **A663**, 951 (2000).

[13] S. Gardner and C. Zhang, Phys. Rev. Lett. **86**, 5666 (2001).

[14] H. Abele, *et al.*, Phys. Rev. Lett. **88**, 211801 (2001).

[15] K. S. Kuzmin, V. V. Lyubushkin, V. A. Naumov, hep-ph/0408107 (2004).

[16] V. Pascalutsa and M. Vanderhaeghen, Phys. Rev. Lett. **94**, 102003 (2005).

[17] M. Gell-Mann, Physics, **1**, 63 (1964); V. De Alfaro, S. Fubini, G. Furlan, and C. Rossetti, *Currents in Hadron Physics*, North-Holland Publishing Co., 1973.

[18] M. D. Slaughter, Phys. Rev. **C49**, R2894 (1994).

[19] L. H. Thomas, Nature **117**, 514 (1926); Philos. Mag. **3**, 1 (1927); E. P. Wigner, Ann. Math. **40**, 149 (1939); A. D. Martin and T. D. Spearman, *Elementary Particle Physics*, North-Holland, Amsterdam, (1970); M. Jacob and G. C. Wick, Ann. Phys.(N. Y.) **7**, 404, (1959); Y. S. Kim and E. P. Wigner, J. Math. Phys. **31**, 55 (1990); For Wigner rotation usage in a number of branches of physics see S. Baskal and Y. S. Kim, Phys. Rev. E. **66**, 026604 (2002).

[20] M. D. Slaughter, Nuc. Phys. **A740**, 383 (2004); M. D. Slaughter, Nuc. Phys. **A703**, 295 (2002); Milton D. Slaughter, Phys. Rev. **C49**, R2894 (1994); M. D. Slaughter and S. Oneda, Phys. Rev. Lett. **59**, 1641 (1987); S. Oneda, T. Tanuma, and M. D. Slaughter, Phys. Lett. **88B**, 343 (1979); M. D. Slaughter and S. Oneda, Phys. Rev. **D39**, 2062 (1989); S. Oneda and K. Terasaki, Prog. Theor. Phys. Suppl. **82**, 1 (1985); S. Oneda and Y. Koide, *Asymptotic Symmetry and Its Implication in Elementary Particle Physics*, World Scientific,

Singapore, (1991).

[21] M. D. Slaughter, Nuc. Phys. **A740**, 383 (2004).

[22] W. W. Ash *et al.*, Phys. Lett. **24B**, 165 (1967).

[23] S. S. Kamalov and S. N. Yang, Phys. Rev. Lett. **83**, 4494 (1999); S. S. Kamalov *et al.*, Phys. Rev. **C64**, 032201(R) (2001); S. S. Kamalov *et al.*, Nuc. Phys. **A684**, 321c (2001).

[24] C. E. Carlson, Phys. Rev. C **34**, 2704 (1986).

[25] K. Joo *et al.*, Phys. Rev. Lett. **88**, 122001 (2002).

[26] R. Beck *et al.*, Phys. Rev. **C61**, 035204 (2000).

[27] P. Bartsch *et al.*, Phys. Rev. Lett. **88**, 142001 (2002).

[28] P. Stoler, Phys. Rev. Lett. **91**, 172303 (2003).

[29] L. M. Stuart *et al.*, Phys. Rev. **D58**, 032003 (1998).

[30] G. Blanpied *et al.*, Phys. Rev. **C64**, 025203 (2001).

[31] V. V. Frolov *et al.*, Phys. Rev. Lett. **82**, 45 (1999).

[32] C. Mertz *et al.*, Phys. Rev. Lett. **86**, 2963 (2001).

[33] S. Kamalov *et al.*, $\gamma^* N \rightarrow \Delta$ *Transition Form Factors: A New Analyses of Data on* $p(e, \acute{e}p)\pi^0$ *at* $Q^2 = 2.8$ *and* 4.0 $(GeV/c)^2$, Phys. Rev. **64**, 032201(R) (2001); Also see S. S. Kamalov and S. N. Yang, *The* Q^2*-dependence of* $\gamma^* N \leftrightarrow \Delta$ *transition form factors*, Nuc. Phys. **A663&664**, 405c (2000); S. S. Kamalov and S. N. Yang, *Pion Cloud and the* Q^2 *Dependence of* $\gamma^* N \leftrightarrow$ *Transition Form Factors*, Phys. Rev. Lett. **83**, 4494 (1999); S. S. Kamalov *et al.*, *Unitary Isobar (MAID) and Dynamical Models for Pion Electroproduction*, Nuc. Phys. **A684**, 321c (2001).

[34] W. W. Ash *et al.*, *Measurement of the* NN^* *form factor*, Phys. Lett. **24B**, 165 (1967).

[35] D. Drechsel, O. Hanstein, S. S. Kamalov, and L. Tiator, *A unitary isobar model for pion photo- and electro-production on the proton up to* 1 *GeV*, Nuc. Phys. **A645**, 145 (1999).

[36] L. M. Stuart *et al.*, *Measurements of the* $\Delta(1232)$ *transition form factor and the ratio* σ_n/σ_p *from inelastic electron-proton and electron-deuteron scattering*, Phys. Rev. **D58**, 032003 (1998).

[37] S. Rock *et al.*, *Measurement of elastic electron-neutron scattering and inelastic electron-deuteron scattering cross sections at high momentum transfer* Phys. Rev. **D46**, 24 (1992).

[38] F. W. Brasse *et al.*, *Parametrization of the* q^2 *dependence of* $\gamma_\nu p$ *total cross sections in the resonance region*, Nuc. Phys. **B110**, 410 (1976).

[39] V. V. Frolov *et al.*, *Electroproduction of the* $\Delta(1232)$ *Resonance at High Momentum Transfer*, Phys. Rev. Lett. **82**, 45 (1999).

[40] G. Blanpied *et al.*, $N \rightarrow \Delta$ *transition and proton polarizabilities from measurements of* $p(\vec{\gamma}, \gamma)$, $p(\vec{\gamma}, \pi^0)$, *and* $p(\vec{\gamma}, \pi^+)$, Phys. Rev. **C64**, 025203 (2001).

[41] K. Joo *et al.*, Q^2 *Dependence of Quadrupole Strength in the* $\gamma^* p \rightarrow \Delta^+(1232) \rightarrow p\pi^0$ *Transition*, Phys. Rev. Lett. **88**, 122001-1 (2002).

[42] C. Mertz *et al.*, *Search for Quadrupole Strength in the Electroexcitation of the* $\Delta^+(1232)$, Phys. Rev. Lett. **86**, 2963 (2001).

[43] Th. Pospischil *et al.*, *Measurement of the Recoil Polarization in the* $p(\vec{e}, \acute{e}\vec{p})\pi^0$ *Reaction at the* $\Delta(1232)$ *Resonance*, Phys. Rev. Lett. **86**, 2959 (2001).r

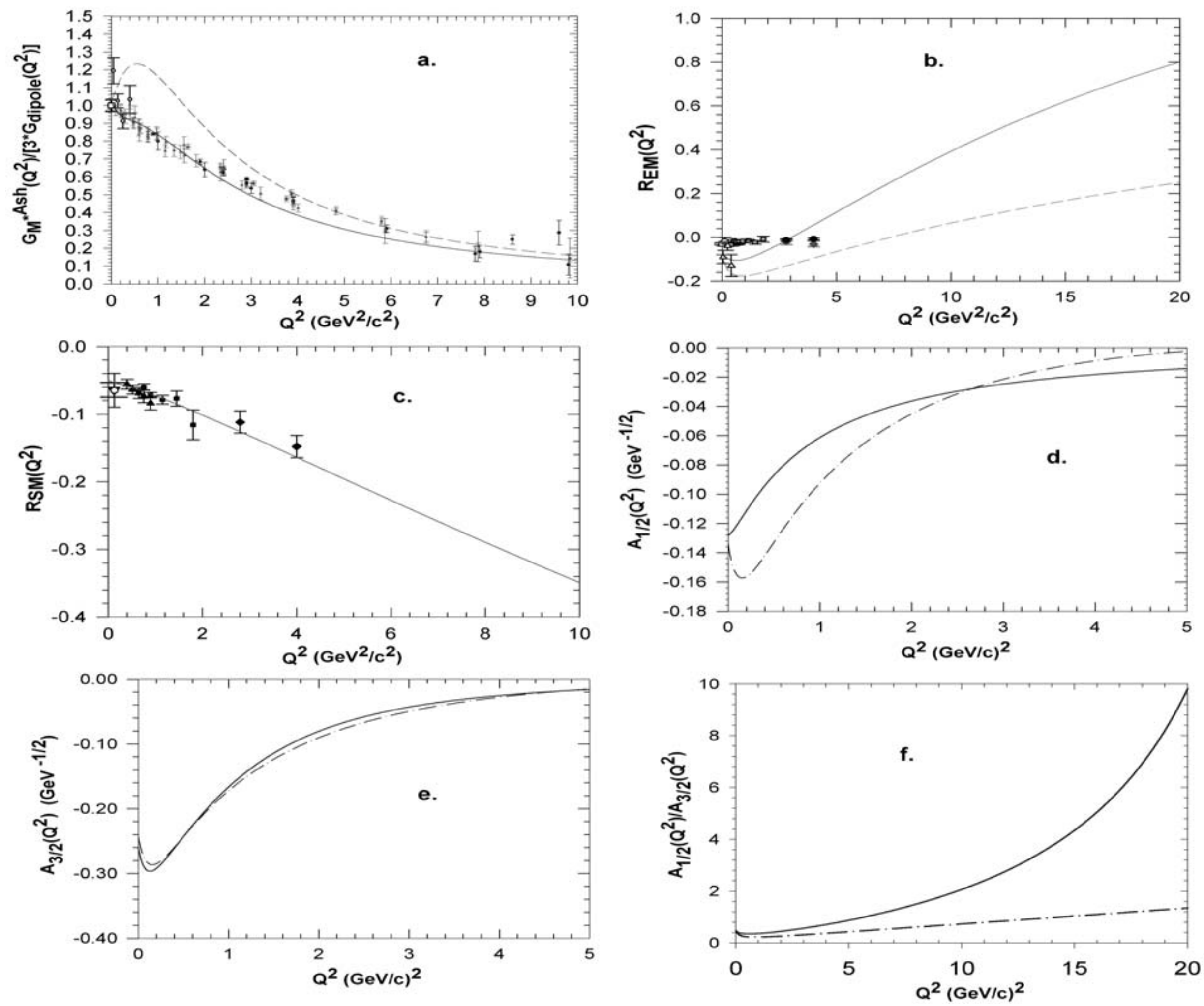

FIG. 1: $G_M^*(Q^2)$ normalized to $3\,G_{\text{dipole}}(Q^2)$, $R_{EM}(Q^2)$, and $R_{SM}(Q^2)$. (a). $G_M^*(Q^2)$ (as defined in [33–35]): Dashed curve: no parameter theoretical calculation and Solid curve: SCC theoretical calculation with a fit to the data of Ref.([33]) with $G_M^*(0)/3\,G_{\text{dipole}}(0) = 1.03$; Open Square data point is from Ref.([34]) where $G_M^*(0)/3\,G_{\text{dipole}}(0) = 1.00$; Circle denoted data is from Ref.([36]); Diamond denoted data is from Ref.([37]); Up-Triangle data is from Ref.([38]); (b). $R_{EM}(Q^2)$: Dashed curve: no parameter theoretical calculation; Solid curve: SCC theoretical calculation with a fit to the data of Ref.([33]) with $R_{EM}(0) = -0.038$. Diamond denoted data is from Ref.([39]); Circle denoted data is from Ref.([40]); Square denoted data are from Ref.([41]); Open-Circle denoted data is from Ref.([42]) (c). $R_{SM}(Q^2)$: Solid curve: SCC theoretical calculation with a fit to the data of Ref.([33]) with $R_{SM}(0) = -0.049$; Diamond denoted data is from Ref.([39]); Down-Triangle denoted data is from Ref.([43]); Circle denoted data is from Ref.([40]); Square denoted data and the Up-Triangle data are from Ref.([41]); Open-Circle denoted data is from Ref.([42]); (d),(e), and (f):The $\Delta N\gamma$ photon decay amplitudes $A_{1/2}(Q^2)$, $A_{3/2}(Q^2)$ and their ratio $A_{1/2}(Q^2)/A_{3/2}(Q^2)$. Solid curves: SCC theoretical calculation with a fit to the data of Ref.([33]). Dash-dotted curves are from an analysis of $p(e,e'p)\pi^0$ data in Ref.([33]). When $Q^2 = 0$, $A_{\frac{1}{2}} = -128 \times 10^{-3} GeV^{-1/2}$ and $A_{\frac{3}{2}} = -260 \times 10^{-3} GeV^{-1/2}$.