Photon-Photon Diffractive Interaction at High Energies

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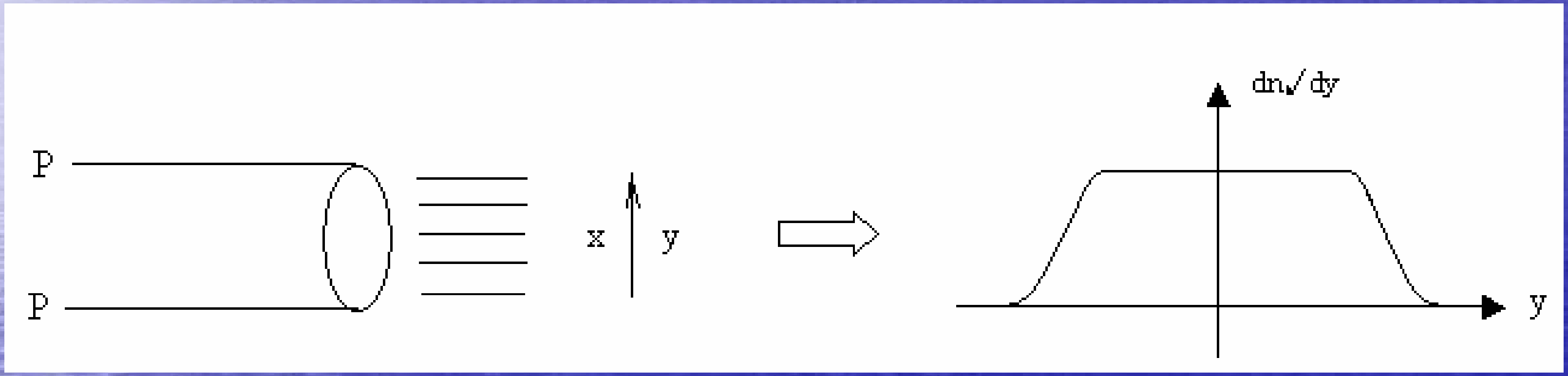
Diffractive Interaction

- Non-Diffractive

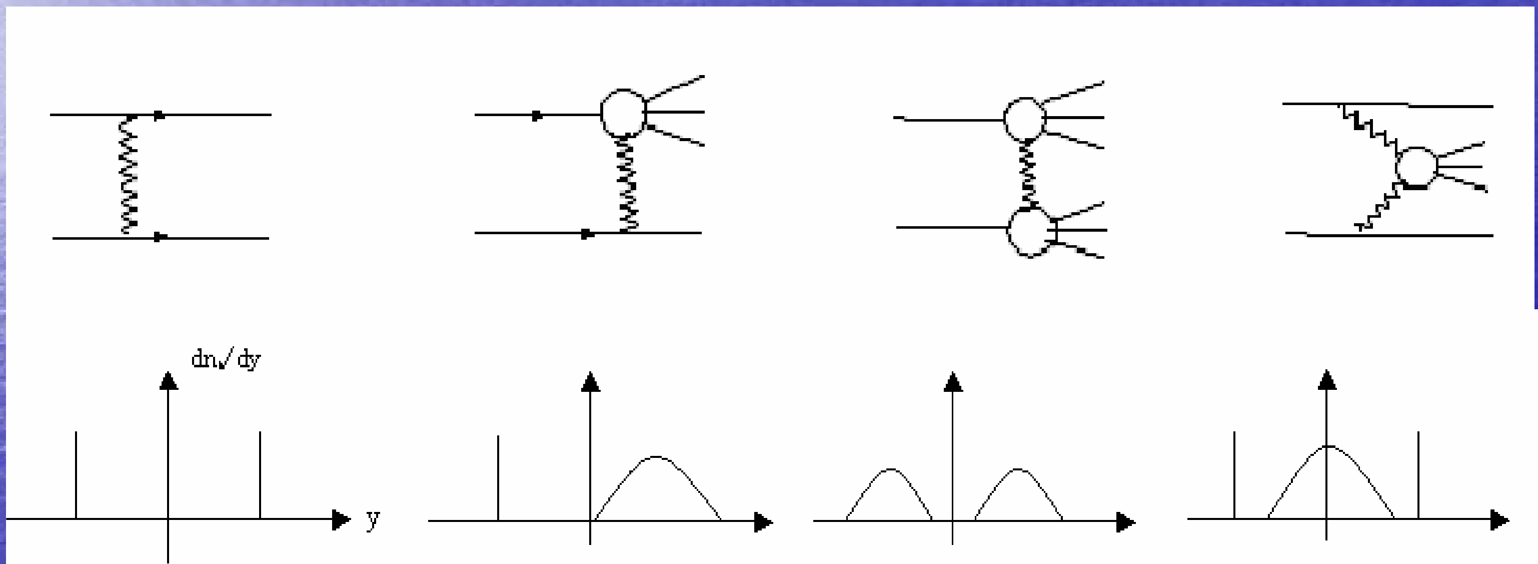


- Rapidity

$$y \equiv \frac{1}{2}\ln\frac{E+P_Z}{E-P_Z} \approx -\ln\tan\frac{\theta}{2} = \eta$$

- e.g. In $pp(\bar{p})$ Diffractive Scattering

Elastic single diffra. double diffra. double P

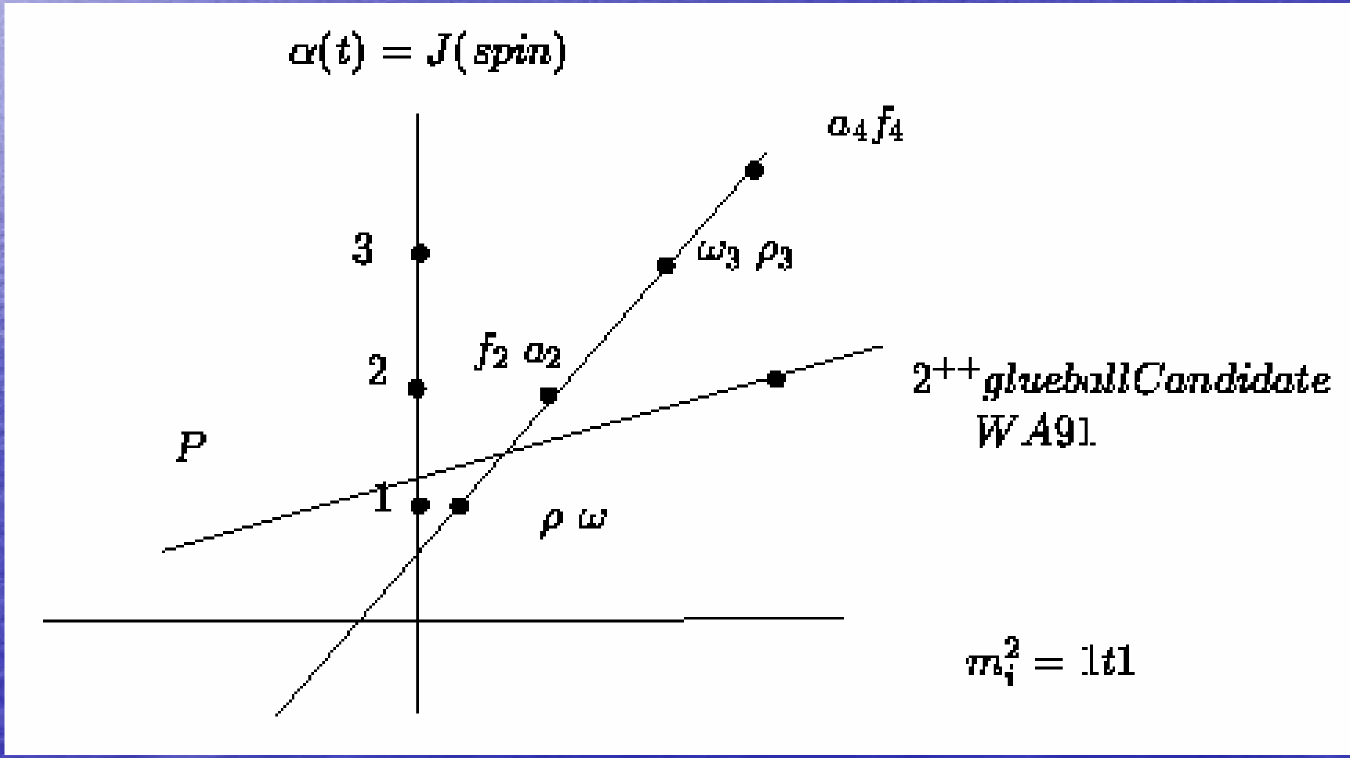


[J.D. Bjorken, 1993]

Regge trajectories

Exchanged Objects: $m_i^2 = |t|$

[Donnachie and Landshoff 1992]



The Single Pole Contribution in Regge Theory

$$\sigma_{tot} \underset{s\to\infty}{\approx} \frac{1}{s} \operatorname{Im} A(s, t=0) \underset{s\to\infty}{\sim} s^{\alpha(0)-1}$$

$$\frac{d\sigma_{el}}{dt} = F(t) s^{2\alpha(t)-2} \underset{t\to 0}{\sim} s^{2\alpha(0)-2}$$

- Regge trajectories have intercepts which do not exceed 0.5. Their exchange leads to total cross sections decreasing, which contradicts to experiment.

- In order to account for the experimental results, the reggeon with intercept larger than one was introduced, named Pomeron, IP

  [Pomeranchuk, 1956; Foldy & Peierls, 1963]

- Pomeron carries the quantum number of the vacuum

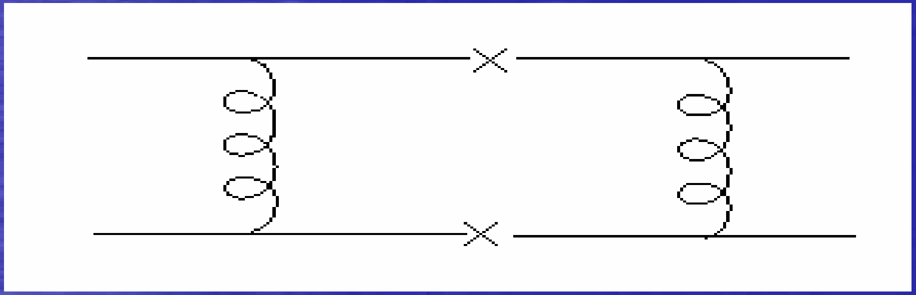
- Is Pomeron IP the glueball ?

QCD Description of Pomeron (BFKL Equation)

[Balitsky, Fadin, Lipatov Kuraev, 1976,1977,1978]

In perturbative QCD, in the limit s >> t and in leading logarithm ln(s/t) approximation (LLA).

e.g. quark-quark scattering



- The gluon is reggized to all orders

$$D_{\mu\nu}(s,q^2) = -i\frac{g_{\mu\nu}}{q^2}\left(\frac{s}{s_0}\right)^{\alpha_g(q^2)-1}$$

$\alpha_g(q^2) = 1 + \varepsilon(q^2)$ is the Regge trajectory of gluon.

- Pomeron emerges as a reggized gluon ladder in color-singlet.

- The leading logarithm result of Pomeron trajectory

$$\alpha_{\mathrm{P}}(t)=1+4\bar{\alpha}_s \ln 2$$

- This is much larger than the phenomenologically obtained intercept of the pomeron

- The leading order (Log) result violate the Froissart bound, that is $Ln^2 s$, at high energy

[Froissart(1961), Martin(1963)]

- NLO corrections to BFKL Kernel greatly reduce the pomeron trajectory

[Fadin and Lipatov, 1998; Ciafaloni and Camici, 1998]

Leading order photon wave function

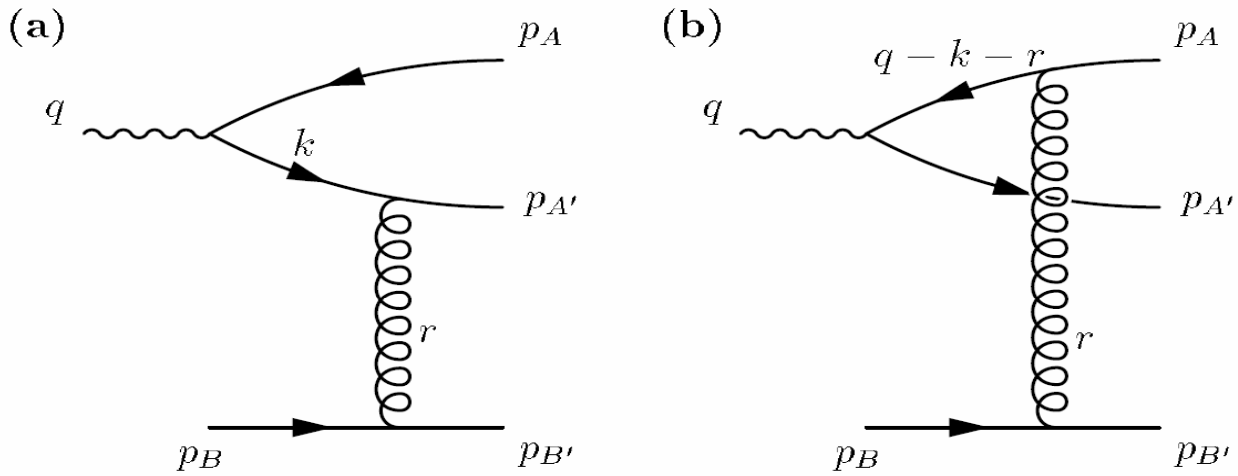


Figure 1: The Feynman diagrams contributing to $\gamma^* + q' \longrightarrow q\bar{q} + q'$ in the high energy limit.

[Gieseke & Qiao 2000;Muller 1990; Nikolaev 1992...]

- Diffractive photon-proton scattering is of particular interest in testing the perturbative QCD, because of not only the HERA prolific experimental data, but also the theoretical development

In Sudakov decomposition

$$k = \alpha q' + \beta p' + k_{\perp}$$

$$r = \tfrac{t}{s} q' + x_P p' + r_{\perp}$$

$$x \equiv x_{Bj} = \frac{-q^2}{2\, p \cdot q} \qquad x_P \equiv \frac{q \cdot (p - p')}{p \cdot q}$$

$$B = \tfrac{x}{x_P} = \frac{Q^2}{Q^2 + M_x^2 - t} \qquad \beta = x_P - \tfrac{x_B t}{S}$$

$$Q^2 = -q^2 \sim \left| r^2 = t \right| \sim M_x^2 << s$$

- In the above limit, the metric tensor decomposes like $g_{\mu\nu} = \frac{2}{s}(p'_{\mu}q'_{\nu} + p'_{\nu}q'_{\mu}) + g^{\perp}_{\mu\nu}$

  for the gluon propagator, only retain the first term, while other terms are suppressed by power of t/s

- The photon polarization vectors can be chosen as

$$\varepsilon^0 = \frac{1}{Q}(q') + x_B p' \qquad \varepsilon(\pm)_{\perp} = \frac{1}{\sqrt{2}}(0, 1, \gamma i, 0)$$

  for longitudinal and transverse ($\gamma = \pm$) cases

- The γP scattering amplitude can be expressed as

$$A = eg^2 \frac{2s}{t} \delta_{\lambda_B \lambda_{B'}} C \sqrt{\alpha(1-\alpha)}(\Psi(k,\alpha) - \Psi(k+r,\alpha))$$

- Here

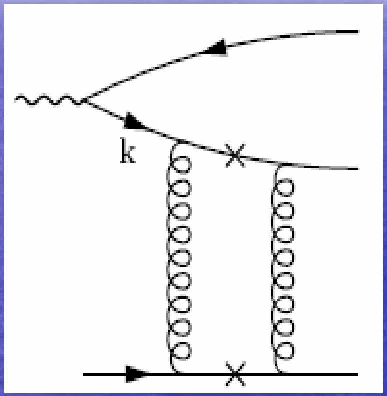
$$\Psi^{0}_{\pm\mp}(k,\alpha)=\frac{2e_{+}\alpha(1-\alpha)Q}{\alpha(1-\alpha)Q^{2}+k^{2}+m^{2}} \qquad \Psi^{\pm}_{\pm\mp}(k,\alpha)=\frac{\sqrt{2}e_{f}\alpha \not{k}}{\alpha(1-\alpha)Q^{2}+k^{2}+m^{2}}$$

$$\Psi^{\pm}_{\mp\pm}(k,\alpha)=\frac{i\sqrt{2}e_{f}m}{\alpha(1-\alpha)Q^{2}+k^{2}+m^{2}} \qquad \Psi^{0}_{\pm\pm}(k,\alpha)=\Psi^{\pm}_{\mp\mp}(k,\alpha)=0$$

$$\Psi^{\pm}_{\mp\pm}(K,\alpha)=\frac{-\sqrt{2}e_{f}(1-\alpha)\not{k}}{\alpha(1-\alpha)Q^{2}+K^{2}+m^{2}} \qquad \not{k}=k_{x}+i\gamma k_{y}$$

The Application of Photon wave function

- To the photon-proton diffractive interaction in two gluon model



$+ \quad \ldots$

- Using Cutkosky rules the full amplitude can be written in "double difference" form

$$A = ieg_s^4 \frac{s}{t} \delta_{\lambda_B \lambda_B'} C' \sqrt{\alpha(1-\alpha)} \int \frac{d^2 l}{(2\pi)^2} \frac{r^2}{l^2 (r-l)^2} \times$$

$$\{\Psi(k,\alpha) + \Psi(k+r,\alpha) - \Psi(k+l,\alpha) - \Psi(k+r-l,\alpha)\}$$

- To include the non-perturbative coupling of the two gluons for the proton, one need to introduce the unintegrated off-diagonal gluon distribution

$$F(x,x',l^2,r^2)$$

and after integrating over l, we would get an off-diagonal gluon distribution

$$\int_0^{Q^2} d^2l F(x,x',l^2,r^2) = G(x,x',l^2,r^2)$$

- In the limit $x \approx x^{'}$, $G(x, x, r^2 = 0, Q^2) = xg(x, Q^2)$

- Therefore, the general amplitude for diffractive scattering off the proton is

$$A = i\frac{\pi}{4} e g_s^4 \sqrt{\alpha(1-\alpha)} \int \frac{d^2 l}{\pi l^2} \frac{r^2}{l^2 (r-l)^2} F(x, x^{'}, l^2, r^2) D\Psi(k, r, l)$$

- Here,

$$D\Psi(k,r,l) = \Psi(k,\alpha) + \Psi(k+r,\alpha) - \Psi(k+l,\alpha) - \Psi(k+r-l,\alpha)$$

- The variable conjugate to k is the transverse separation of the $q\bar{q}$ pair, the ρ , which is called the “dipole size”

- The variable conjugated to the momentum transfer between the diffractive system and the proton is the impact parameter b

- The conjugated photon wave functions are

$$\Psi^{0}_{\pm\mp}(\rho,\alpha)=\frac{1}{\pi}e_f\alpha(1-\alpha)QK_0(\delta\rho)$$

$$\Psi^{\pm}_{\pm\mp}(\rho,\alpha)=\frac{i}{\pi}e_f\alpha\delta\frac{\rho\cdot\varepsilon}{\rho}K_1(\delta\rho)$$

$$\Psi^{\pm}_{\mp\pm}(\rho,\alpha)=-\frac{i}{\pi}e_f(1-\alpha)\frac{\rho\cdot\varepsilon}{\rho}K_1(\delta\rho)$$

$$\Psi^{\pm}_{\pm\pm}(\rho,\alpha)=\frac{im}{2\pi}e_fK_0(\delta\rho)$$

$\delta^2=\alpha(1-\alpha)Q^2+m^2;\quad K_\nu(z)$ is the modified Bessel function

- The diffractive amplitude configuration space is

$$\tilde{A}^D(\rho,b)=\int\frac{d^2k}{(2\pi)^2}\int\frac{d^2r}{(2\pi)^2}e^{ik\cdot s}e^{ir\cdot b}A^D(k,r)$$

$$=B\Psi(\rho,\alpha)\int\frac{d^2r}{(2\pi)^2}e^{ir\cdot b}\int\frac{d^2l}{\pi l^2}\alpha_s(\mu^2)F(x,x^{'},l^2,r^2)\times$$

$$[1-e^{-il\cdot s}][1-e^{-i(r-l)\cdot s}]$$

$$=\Psi(s,\sigma)\sigma_{q\bar{q}}(\rho,b)$$

- The dipole picture of diffractive interaction

NLO Photon Impact Factor

- People believe that at LCs things would be much clear
- To make full NLO prediction, we have the NLO BFKL Kernel, but no NLO Photon impact factor yet

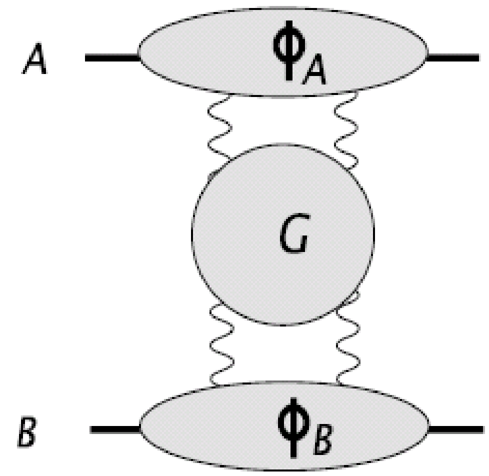


Fig.1: the elastic high energy scattering process $AB \to AB$.

- To calculate the NLO Impact factor, we can use $\gamma^* q \to (q\bar{q})q$ scattering as the starting point, in the high energy limit

$$t, Q^2, t_a, t_b, M^2 << S$$

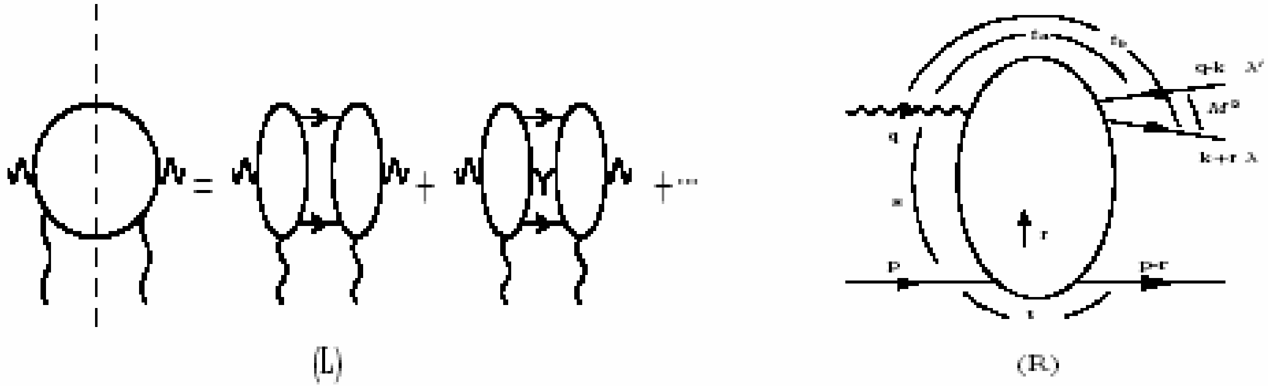


FIGURE 3. (L)Contributions to the photon impact factor; (R)Kinematics of the process $\gamma^* + q \to q\bar{q} + q$.

- Taking the Regge ansatz for the scattering amplitude

$$A = \Gamma^{a}_{\gamma^* \to q\bar{q}} \frac{s}{t}[(\frac{s}{-t})^{\omega} + (\frac{s}{-t})^{\omega}]\Gamma^{a}_{qq}$$

- $1+\omega$ is the gluon trajectory. Expanding all terms in powers on the strong coupling g_s

$$\omega = g_s{}^2\omega^{(1)} + g_s{}^4\omega^{(2)}$$

$$\Gamma^{a}_{\gamma^* \to q\bar{q}} = g_s \Gamma^{(0),a}_{\gamma^* \to q\bar{q}} + g_s{}^3 \Gamma^{(1),a}_{\gamma^* \to q\bar{q}}$$

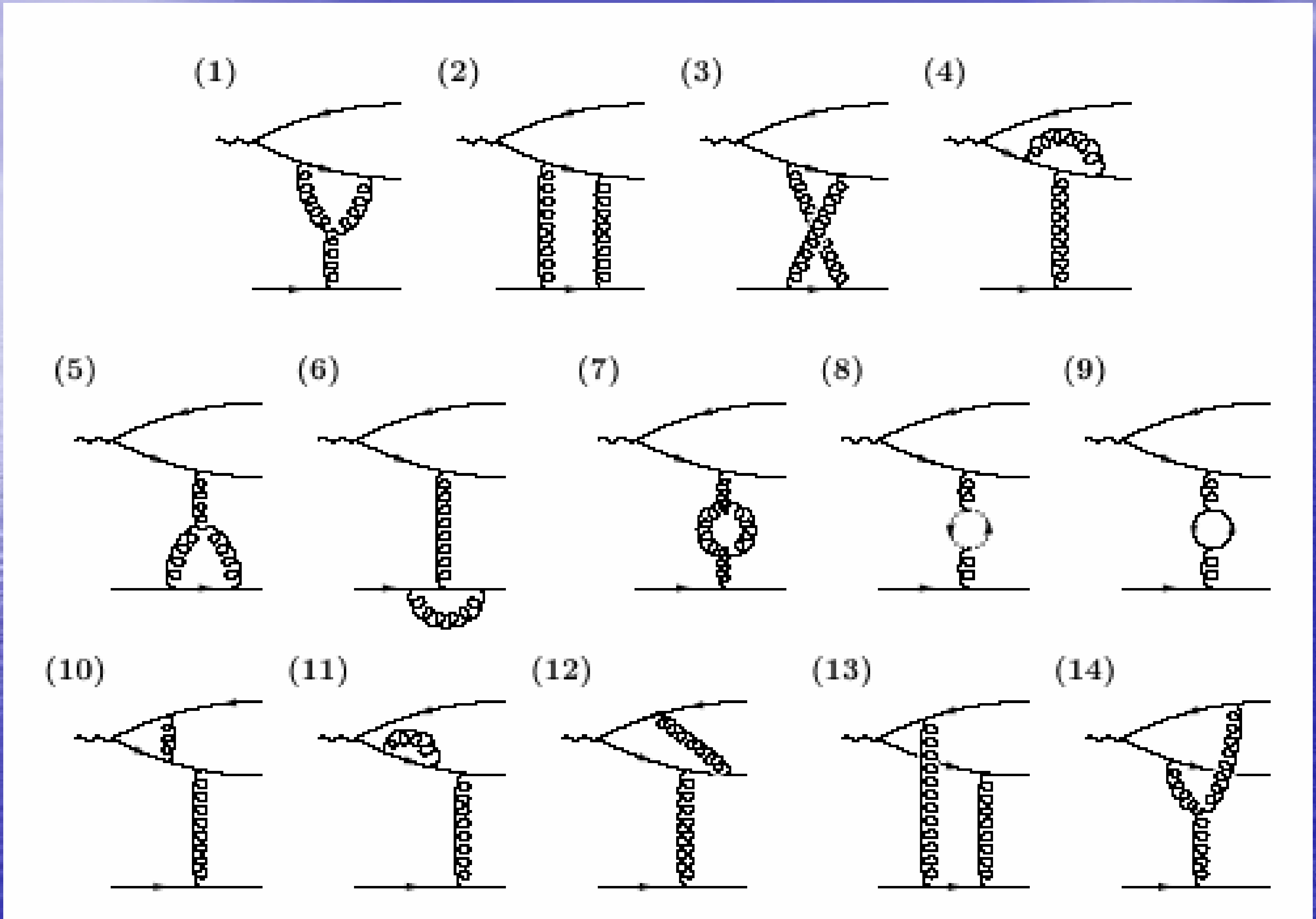
$$\Gamma^{a}_{qq} = g_s \Gamma^{(0),a}_{qq} + g_s{}^3 \Gamma^{(1),a}_{qq}$$

- Then $T = g_s^2 T^{(0)} + g_s^4 T^{(1)}$ [Bartels,Gieseke, Qiao,2001]

$$T^{(0)} = \Gamma^{(0),a}_{\gamma^* \to q\bar{q}} (-) \frac{2s}{t} \Gamma^{(0),a}_{qq}$$

And
$$T^{(1)} = \Gamma^{(1),a}_{\gamma^* \to q\bar{q}} (-) \frac{2s}{t} \Gamma^{(0),a}_{qq} + \Gamma^{(0),a}_{\gamma^* \to q\bar{q}} (-) \frac{2s}{t} \Gamma^{(1),a}_{qq}$$
$$+ \Gamma^{(0),a}_{\gamma^* \to q\bar{q}} (-) \frac{s}{t} \Gamma^{(0),a}_{qq} \omega^{(1)} [\ln \frac{s}{-t} + \ln \frac{-s}{-t}] \Gamma^{(0),a}_{qq}$$

- The calculation of virtual correction to $\Gamma^{(1),a}_{\gamma^* \to q\bar{q}}$ involving 14 diagrams


(1)
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(14)

- In practical calculation, we have used the computer algebra system Mathematica with package Feyncalc

- Using the loop integral technique given by [Bern, Dixon and Kosower,1994],all the results are presented analytically. Finally, we need to take the high energy limit

- It has been showed that the ultra-violet divergencies are removed in $\overline{MS}$ scheme; the infrared divergencies are cancelled out by adding real corrections [Bartels, Gieseke and Kyrieleis, 2002]

Photon-Photon Diffractive Scattering

γ * γ * diffractive scattering cross section can be expressed as

$$\sigma_{AB} = \int \frac{d^2\boldsymbol{r}}{(2\pi)^2} \int \frac{d^2\boldsymbol{r}'}{(2\pi)^2} \int \frac{d\omega}{2\pi i} \left(\frac{s}{s_0}\right)^{\omega} \Phi_A \frac{1}{\boldsymbol{r}^2} G_{s_0}(\boldsymbol{r}, \boldsymbol{r}'; \omega) \frac{1}{\boldsymbol{r}'^2} \Phi_B ,$$

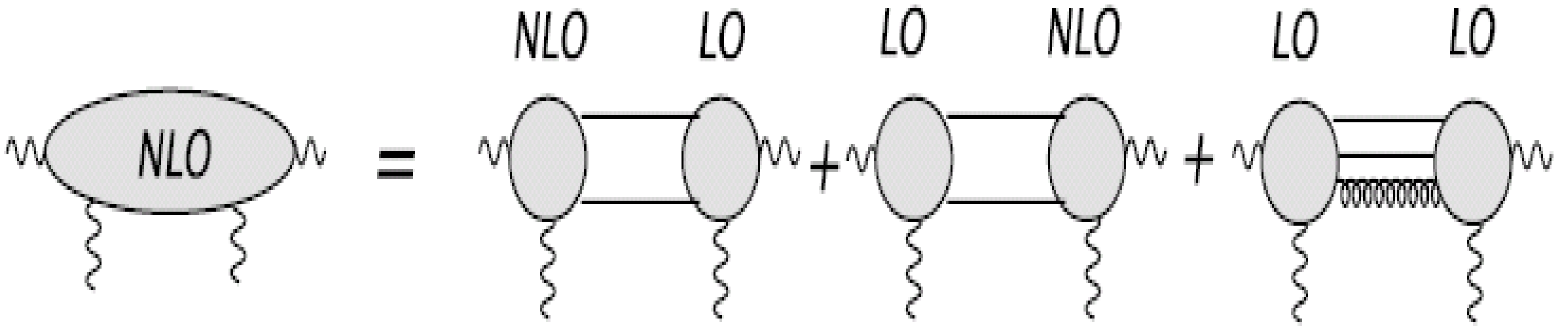


Fig.2: quark-antiquark (virtual) and quark-antiquark-gluon (real) contributions to the NLO impact factor.

γ * γ * diffractive scattering cross section up to order α_s^3

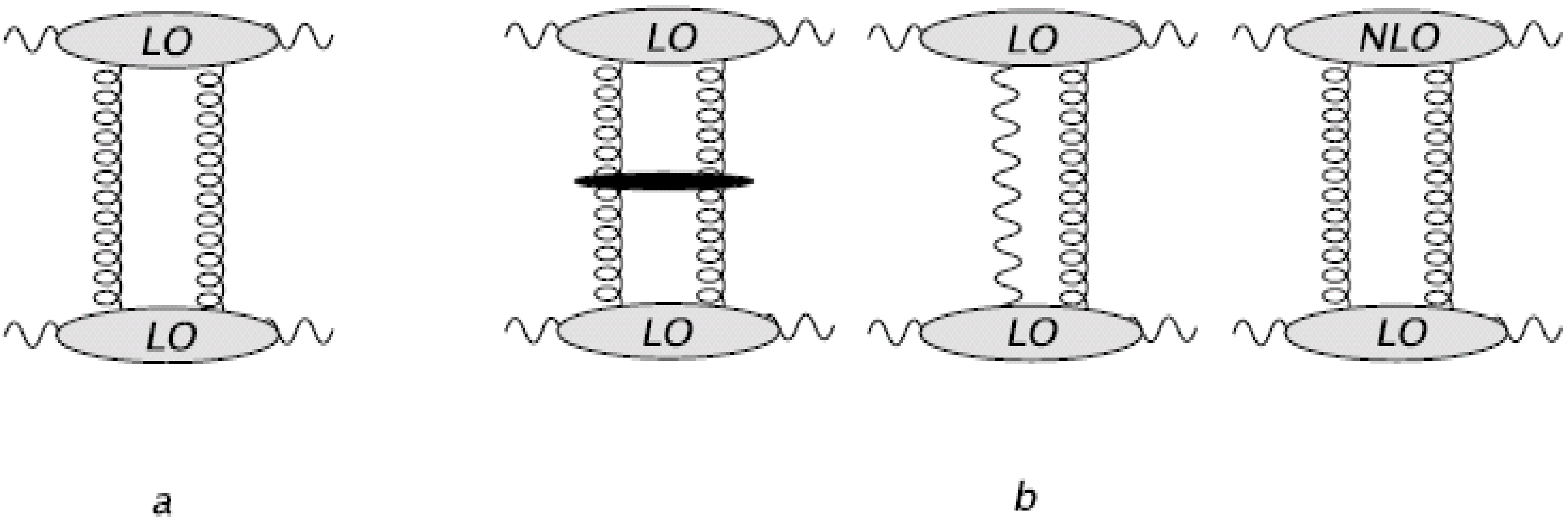


Fig.3: $\gamma^*\gamma^*$ scattering at fixed order α_s:
(a) order α_s^2; (b) α_s^3: in the second diagram, one of the gluons (left) is reggeized, all other t-channel gluons are elementary.

γ * γ * diffractive scattering cross section at order α_s^3

$$\sigma_{AB}^{(1)} = \frac{1}{s}\mathrm{Im}\, T_{AB}^{(1)}(s, t=0)$$

$$= \ln(s/s_0)\int \frac{d^{D-2}\boldsymbol{r}}{(2\pi)^{D-2}}\frac{d^{D-2}\boldsymbol{r}'}{(2\pi)^{D-2}}\Phi_A^{(0)}\frac{1}{\boldsymbol{r}^4}\mathcal{K}(\boldsymbol{r},\boldsymbol{r}')_{real}\frac{1}{\boldsymbol{r}'^4}\Phi_B^{(0)}$$

$$+ \int \frac{d^{D-2}\boldsymbol{r}}{(2\pi)^{D-2}}\Phi_A^{(0)}\ln(s/\boldsymbol{r}^2)2\omega^{(1)}(\boldsymbol{r}^2)\frac{1}{\boldsymbol{r}^4}\Phi_B^{(0)}$$

$$+ \int \frac{d^{D-2}\boldsymbol{r}}{(2\pi)^{D-2}}\Phi_A^{(1)}\frac{1}{\boldsymbol{r}^4}\Phi_B^{(0)} + \int \frac{d^{D-2}\boldsymbol{r}}{(2\pi)^{D-2}}\Phi_A^{(0)}\frac{1}{\boldsymbol{r}^4}\Phi_B^{(1)}.$$

Photon Impact Factor at NLO

$$\Phi_{\gamma^*}^{(1)} = \Phi_{\gamma^*}^{(1,virtual)}\Big|^{finite} - \frac{2\Phi_{\gamma^*}^{(0)}}{(4\pi)^2}\left\{\beta_0 \ln\frac{\boldsymbol{r}^2}{\mu^2} + C_F \ln(\boldsymbol{r}^2)\right\}$$

$$+ \frac{1}{(4\pi)^2}\int d\boldsymbol{k}\int_0^1 d\alpha\, \mathcal{I}_2(\alpha,\boldsymbol{k})\Big\{C_A\left[\ln^2\alpha(1-\alpha)s_0 - \ln^2 M^2 - 2\ln\boldsymbol{r}^2\ln\frac{s_0}{\boldsymbol{r}^2}\right]$$

$$+ 2C_F\left[8 - 3\ln\alpha(1-\alpha)\Lambda^2 + \ln^2 M^2 + \ln^2\frac{\alpha}{1-\alpha}\right]\Big\}$$

$$+ C_A\,\Phi_{\gamma^*}^{(1,real)}\Big|_{C_A}^{finite} + C_F\,\Phi_{\gamma^*}^{(1,real)}\Big|_{C_F}^{finite}.$$

- Because of the complexity, the NLO corrections to photon impact factor are only presented numerically

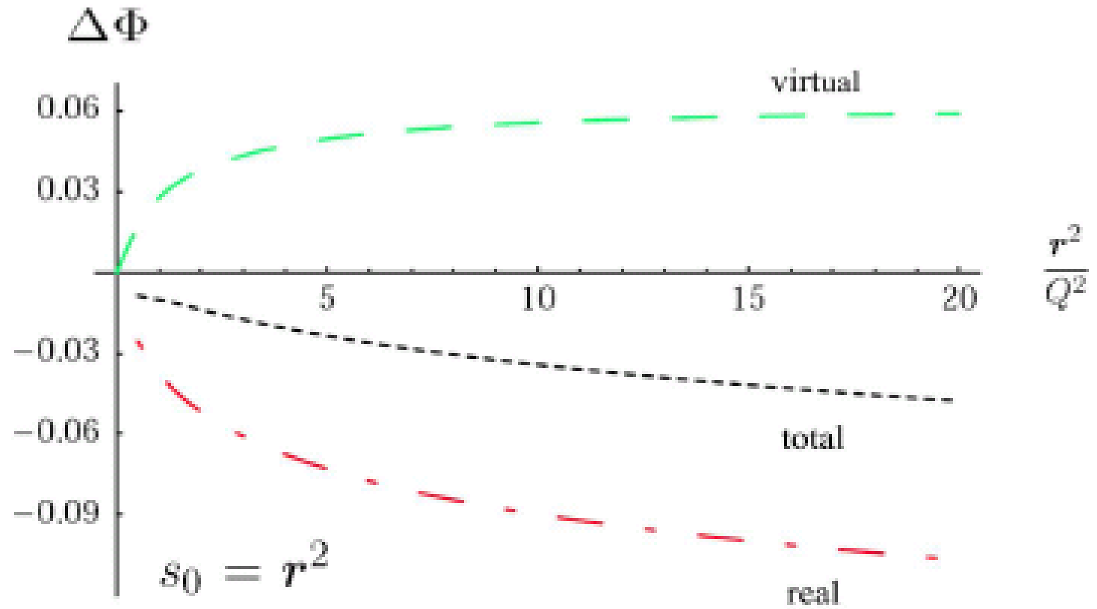
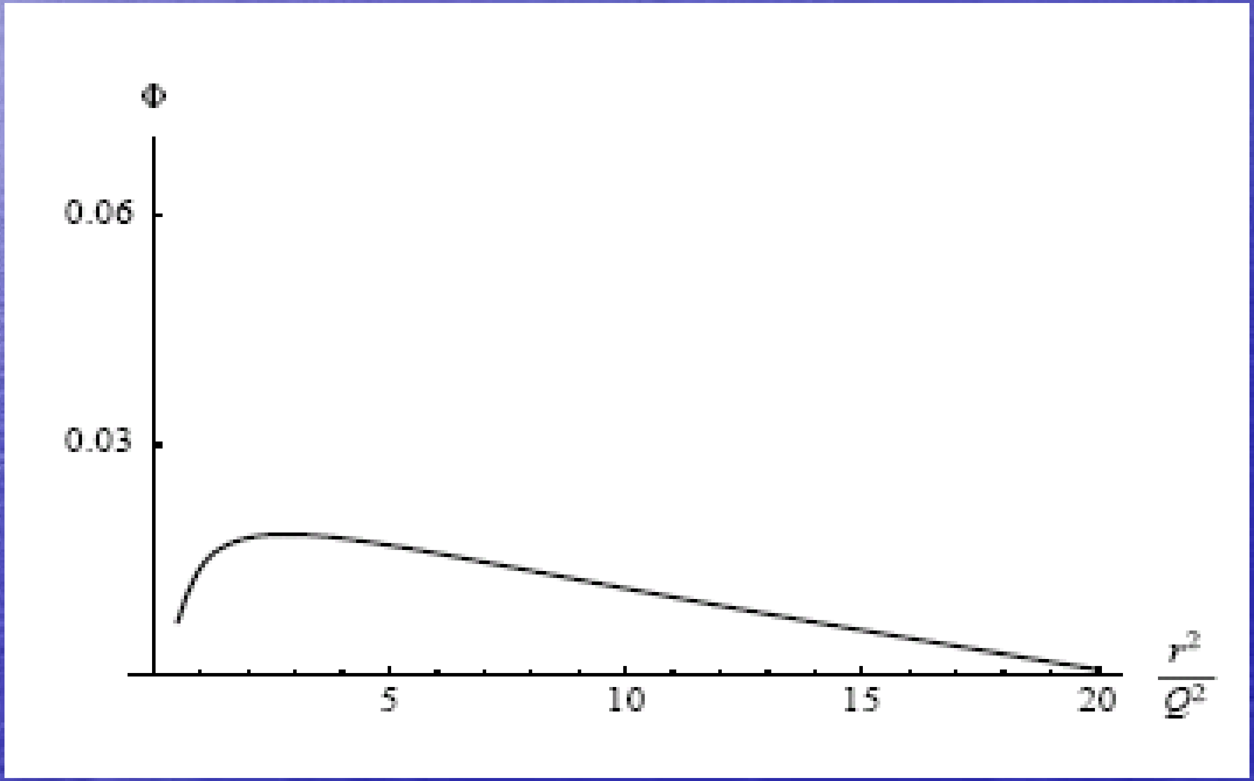


Figure 3: Virtual, real and total NLO corrections to the photon impact factor for $s_0 = r^2$.

- The full next-to-leading order photon impact factor at $S_0 = |\vec{r}|^2$



- To check the numerical result, one should get the s_0 independence of NLO γ*γ* total cross section

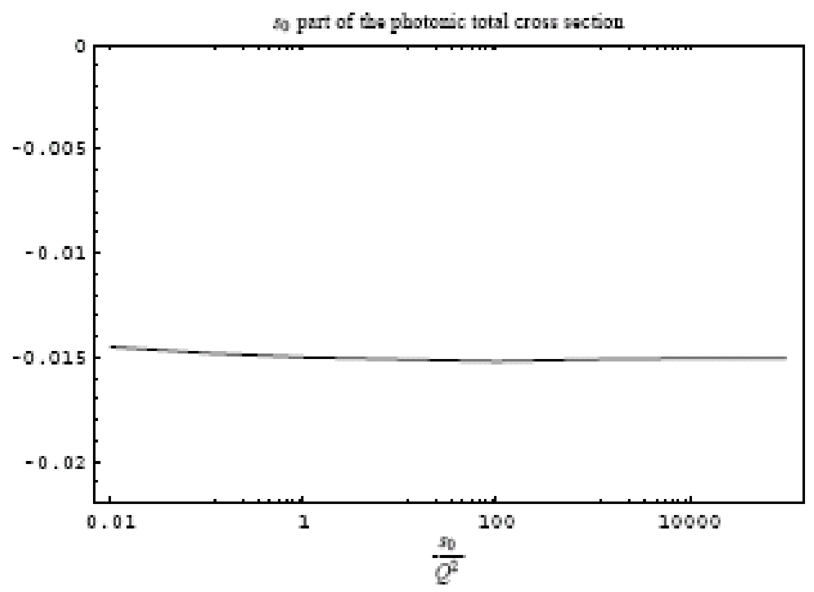


Figure 2: The part of the total $\gamma^*\gamma^*$ cross section that depends on s_0, plotted as a function of s_0/Q^2.

Summary & Outlook

- The full fixed order (α_s^3) prediction for ɣ * ɣ * total cross-section well come soon
- [Bartels, Chachamis and Qiao, in preparation]
- In the future, our NLO photon impact factor should convoluted with NLO BFKL Green's function in making predictions

- The ILC provide us a very good play ground for testing and understanding the QCD. The diffractive interaction would be one of the emphases

- So far, the meaning of the NLO calculation for hard diffractive interaction is more on the theoretical side, rather than for the phenomenological sense

Thank you for your attention