

Curved space, monsters and black hole entropy

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The meaning of entropy

Entropy = log of number of microstates consistent with some macro condition.

$$S \sim \ln \{\text{\# of microstates } N\}$$

Typically, entropy is *extensive*:

$$\ln(c^V) = V \ln c$$

The dimensionality of the Hilbert space describing a volume V is $\dim \mathcal{H} = c^V$ ($c = 2$ for qubit). $S \sim$ number of d.o.f.

Black hole entropy: $S = A/4 \rightarrow \exp(A/4)$ microstates.

Entropy bounds, holography, microstates

Gedanken experiment: Start with arbitrary matter system of size R and entropy S , smash into black hole of area A .

By Second Law of thermodynamics, obtain entropy bound

$$S < A$$

Hilbert space of a d dimensional system (with gravity) is only that of a $d - 1$ dimensional (non-gravitating) system: **AdS/CFT duality**.

String theory calculations (Strominger and Vafa) suggest black hole entropy is due to individual microstates (D-brane configurations).

'tHooft bound

Exclude states whose energies are so large that they would have already caused gravitational collapse: $E < R$ (Hoop Conjecture)

Compute entropy; dominated by thermal configurations at large V :

$$S \sim T^3 R^3 \quad , \quad E \sim T^4 R^3$$

$E < R$ then implies

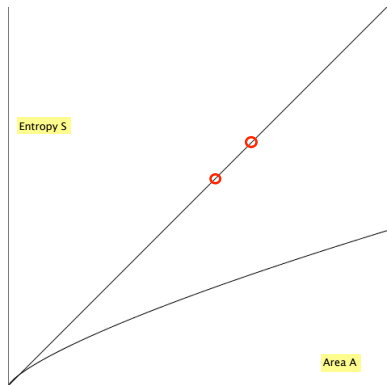
$$T \sim R^{-1/2} \quad \rightarrow \quad S < R^{3/2} \sim A^{3/4}$$

$$S < A^{3/4}$$

Note: Black hole *density* decreases with size. For any constant density $E(R) > R$ for sufficiently large R !

'tHooft bound

Ordinary matter satisfies $S < A^{3/4}$. What does this say about black holes, which have $S \sim A$? **There is an entropy gap!**



The number of ways to form a given black hole is overwhelmingly dominated by simply adding some energy to a slightly smaller hole.

Negative binding energy

Consider N particles of mass m . In GR there is negative binding energy $\equiv \Delta$. The ADM mass can be less than Nm :

$$M = Nm - \Delta$$

In fact, one can achieve (Zeldovich and Novikov; ADM; ...)

$$\frac{M}{Nm} = \frac{Nm - \Delta}{Nm} \ll 1 .$$

This suggests that entropy to mass ratios can be very high. If the object becomes a black hole its area will scale as $A \sim M^2$.

Can the entropy equal or exceed A ?

Curved space

Goal: Generalize 'tHooft analysis to curved space.

Construct configurations with **large proper volume** (fixed entropy density; large total entropy) but **small ADM mass**.

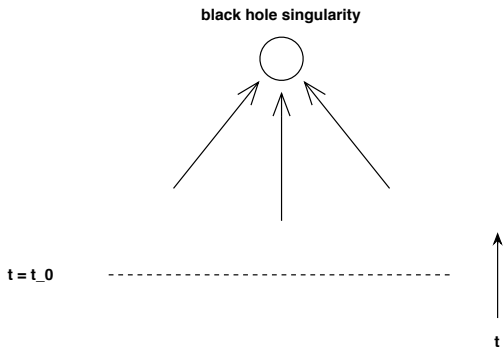
Configurations will be static (macroscopic moment of time symmetry; time reversal invariance) and satisfy Einstein constraints.

They comprise good initial data (matter + metric) for evolution using Einstein equations.

Collapse

Each configuration satisfies the Einstein constraints and, as we will see, collapses to form a black hole.

Each leads to a *distinct* black hole internal state; our goal is to count them.



Curved space: key formulae

Total ADM mass (includes negative binding energy):

$$M \equiv M(R) = 4\pi \int_0^R dr' r'^2 \rho(r') ,$$

Total entropy: ($\epsilon(r) = 1 - 2M(r)/r$)

$$S = 4\pi \int_0^R dr r^2 \epsilon(r)^{-1/2} s(r) .$$

Both $\rho(r)$ and $s(r)$ are proper densities (as seen by stationary inertial observer).

E.g., **thermal photons**: $\rho(r) \sim T(r)^4$, $s(r) \sim T(r)^3$ or $s(r) \sim \rho(r)^{3/4}$.

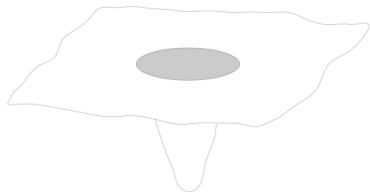
Maximize S while holding M fixed.

Curved space: monster

The *proper* volume of our object is

$$V_p = 4\pi \int_0^R dr r^2 \epsilon(r)^{-1/2} . \quad (1)$$

Fixed ADM mass (surface area); potentially infinite proper volume and total entropy, keeping entropy and energy *densities* finite.

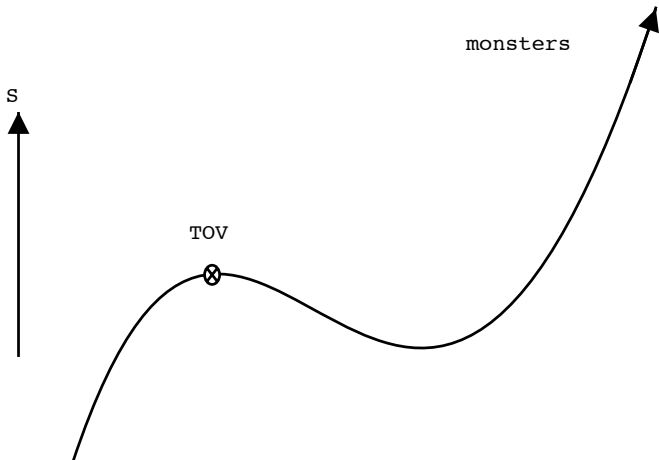


Trick: adjust $\epsilon(r) = 1 - 2M(r)/r \approx 0$
in some large region.

See also Sorkin, Wald and Zhang
(1981)

Entropy extrema

Hold ADM mass M fixed, extremize entropy S .



Example monster: blob

Suppose: small core of radius r_0 , mass M_0 and density profile

$$\rho(r) = \rho_0 \left(\frac{r_0}{r} \right)^2 \quad (r_0 < r < R) . \quad (2)$$

Then

$$M(r) = M_0 + 4\pi\rho_0 r_0^2 (r - r_0) . \quad (3)$$

Let $8\pi\rho_0 r_0^2 = 1$ so that

$$\epsilon(r) = \epsilon_0 \left(\frac{r_0}{r} \right) , \quad (4)$$

where $\epsilon_0 = 1 - 2M_0/r_0$.

Example monster

Total entropy (neglecting the small core region $r < r_0$):

$$S \sim 4\pi \int_{r_0}^R dr r^2 \left(\frac{r}{r_0 \epsilon_0} \right)^{1/2} \rho^{3/4} \sim \frac{\rho_0^{3/4} r_0}{\sqrt{\epsilon_0}} R^2 . \quad (5)$$

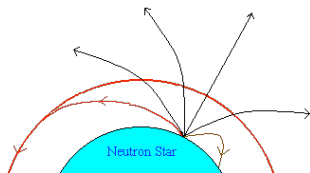
- 1) Area scaling has been achieved.
- 2) S can be made as large as desired by taking ϵ_0 small.
- 3) Can obtain faster than A scaling by taking $\epsilon(r)$ to approach zero faster than $1/r$.

Escape angle and the fate of monsters (skip)

Trapped surface? No.

Horizon? Eventually, yes.

Future of monster interior does not include future null infinity \mathcal{I}^+

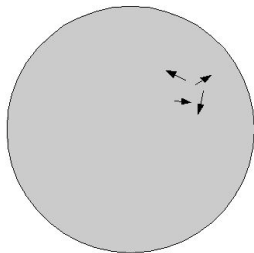


Critical escape angle $\equiv \theta_c$.
 $\theta_c^2 \sim \epsilon(r) = 1 - 2M(r)/r$.

Escape angle and the fate of monsters (skip)

All monsters must have $\epsilon \approx 0$ (and $\theta_c \approx 0$) in large subregion.

Hence, will inevitably evolve into a black hole: mean-field analysis shows net inward energy flow.



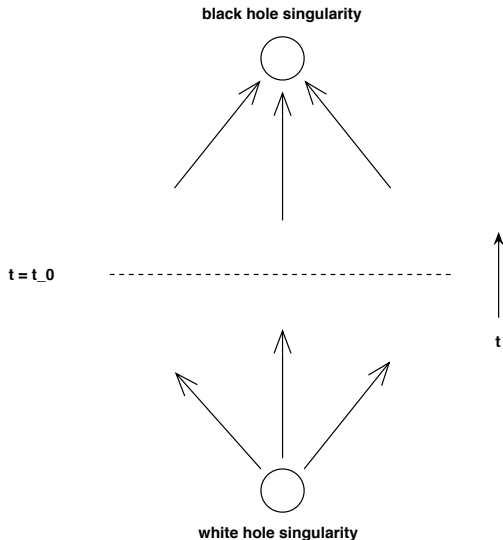
Will also evolve into a black hole if *time-reversed*.

So, cannot form from good initial data (without intervention): requires initial *white hole* singularity.

Isolated monster

Matter ejected from white hole reaches turning point (object is gravitationally bound) and recollapses into black hole.

Note time-reversal invariance.



Build a monster

Assume arbitrarily advanced civilization, constraints only from fundamental physics. **Can one construct a monster (avoiding the initial white hole singularity)?**

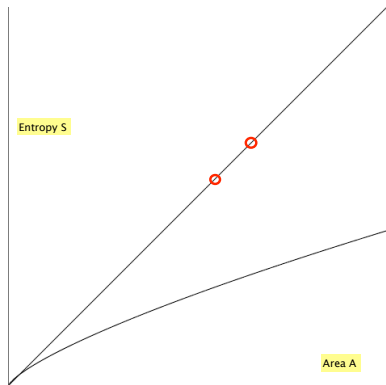
Buildability condition: require configuration to be no closer than a thermal wavelength $\lambda \sim \rho^{-1/4}$ from its Schwarzschild radius.

$$r\epsilon(r) > r - 2M(r) > \lambda$$

implies the bound $S < A$.

Can't build monsters with entropy $S > A$.

Entropy gap?



Perhaps this closes the entropy gap?

Most entropic pre-black hole state is **either** a slightly smaller black hole, or a monster state with entropy A .

Build a monster: tunneling

Initial data: collapsing spherical shell of energy, ADM mass M , other quantum numbers $Q = J = 0$.

Same quantum numbers as monster.

There must be a **nonzero probability** for our initial data to evolve (tunnel) into the monster state, even those with $S \gg A!$

Otherwise, \exists new selection rule forbidding certain transitions between states with same quantum numbers.



How big is the Hilbert space for gravity?

Let Φ = matter fields; g = geometry or metric.

$$(\Phi, g) \in \mathcal{H}$$

Good semiclassical evidence for

$$\dim \mathcal{H} \gg \exp A$$

Does black hole entropy count the number of possible microstates of the hole? # of microstates $\sim \exp A/4$

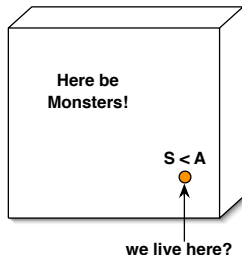
Problematic if tunneling to monster states is allowed: a monster-like configuration might lurk behind the horizon, with the possibility of many more microstates.

Conclusions

Using curved space, one can (mathematically) construct objects with more entropy than a black hole of equal mass.

Conventional interpretation of black hole entropy and AdS/CFT suggests that configurations with $S \gg A$ should be **eliminated** from the Hilbert space of quantum gravity.

Hilbert space of quantum gravity?



Black hole entropy

A mystery of modern physics: $S_{\text{BH}} = A/4$

Entropy given by area in Planck units. One bit per Planck area.

Bekenstein-Hawking: $T \sim R^{-1}$, $R \sim M$

$$S_{\text{BH}} \sim \int \frac{dQ}{T} \sim \int R dM \sim R^2$$

Black hole microstates?

String theory calculations (Strominger and Vafa) suggest black hole entropy is due to individual microstates (D-brane configurations).

Number of configurations is

$$N \sim \exp A/4$$

End of story?

Curved space

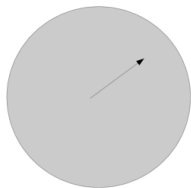
Consider spherically symmetric, but not necessarily static, distributions of matter

$$ds^2 = -g_{tt}(r, t)dt^2 + g_{rr}(r, t)dr^2 + r^2 d\Omega^2 . \quad (6)$$

Energy within radius r :

$$M(r) = 4\pi \int_0^r dr' r'^2 \rho(r') , \quad (7)$$

where $\rho(r) = \rho(r, t_0)$ is the proper energy density (i.e., as seen by a stationary observer at r) on the initial time slice $t = t_0$.



ADM mass: $M \equiv M(R)$, where R is radius of object.

Curved space

Define

$$\epsilon(r) = 1 - \frac{2M(r)}{r} , \quad (8)$$

Then, assuming the matter to be initially at rest w.r.t. our (r, θ, ϕ) coordinates, the metric on that slice is fully determined by

$$g_{rr}(r, t_0) = \epsilon(r)^{-1} . \quad (9)$$

Each choice of $\rho(r)$ yields a good initial configuration.

Curved space

Entropy: assume covariantly conserved entropy current

$j^\mu : j^\mu{}_{;\mu} = 0$. Stokes theorem:

$$S_\Sigma = \int_\Sigma d^3x \sqrt{\gamma} s = \text{constant} , \quad (10)$$

where integral is over a constant time slice Σ with induced metric γ and unit normal $n^\mu \sim (\partial_t)^\mu$.

$s = -j^\mu n_\mu$ is the proper entropy density (as seen by a stationary inertial observer). In our coordinates, $s(r) = j^0(r, t_0) g_{tt}(r, t_0)^{1/2}$.

Total entropy on the initial time slice t_0 is

$$S = 4\pi \int_0^R dr r^2 \epsilon(r)^{-1/2} s(r) . \quad (11)$$

Black hole entropy and microstates

It is claimed that black hole entropy counts the number of possible microstates of the hole (Strominger and Vafa):

$$\# \text{ of microstates} \sim \exp A/4$$

This is problematic if tunneling to monster states is allowed: a monster-like configuration might lurk behind the horizon, with the possibility of many more microstates.

Singularity and selection rule

Consider two states with **identical** quantum numbers:

$|M\rangle$: monster with entropy $> A$

$|\Psi\rangle$: smooth initial data which evolves to a black hole

No tunneling: $\langle M|e^{-iHt}|\Psi\rangle = 0 \quad \forall t$

Evolve both **backwards** in time: $e^{iHT}|M\rangle$ is a singular geometry (**white hole**) while $e^{iHT}|\Psi\rangle$ is a smooth geometry, for some choice of $T > 0$.

Perhaps

$$\langle M|e^{-iHt}|\Psi\rangle = \langle M|e^{-iHT} e^{-iHt} e^{+iHT}|\Psi\rangle = 0 ,$$

justifying the selection rule. (**Many worlds** issues lurk here.)

AdS/CFT

It is plausible that monster configurations exist in AdS.

Can they be described by the boundary CFT? **Not enough degrees of freedom!**

Related discussion: inflationary pocket universes in AdS/CFT

Freivogel, Hubeny, Maloney, Myers, Rangamani and Shenker, JHEP 0603:007,2006.

Conclusion: new selection rule in quantum gravity!