Modeling the $Q\bar{Q}$ Flux Tube on the Lightcone Worldsheet

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1 Introduction

The utility of string theory in understanding the heavy $Q\bar{Q}$ system in large N gauge theory.

$$\text{String Theory} : \sum \text{Planar Diagrams}$$
$$::$$
$$\text{Bethe-Salpeter Equation} : \sum \text{Ladder Diagrams}$$

- Free "string theory" gives large N limit of a local underlying matrix theory.
- $1/N$ is the key to "Field/String Duality"
- $1/N$ points to more fundamental formulation of string theory.
- QCD is a good arena to explore string theory because it is well understood both experimentally and theoretically.
- Bonus: deeper theoretical understanding of quark confinement may emerge.

Field/String Duality is more general than AdS/CFT
The essential point seems to be $N \to \infty$:

- At $N = \infty$ perturbation theory is sum of only planar diagrams, suggesting the worldsheet of a string description for *any* $N \times N$ matrix field theory.
- The AdS/CFT correspondence is most sharply formulated in the $N \to \infty$ limit.
- In the regime $g_s^2 N \gg 1$ the AdS string is accurately described semi-classically.
- QCD has no coupling to tune (dimensional transmutation), so its dual string must be inherently quantum.

Lightcone Worldsheet vs. AdS/CFT

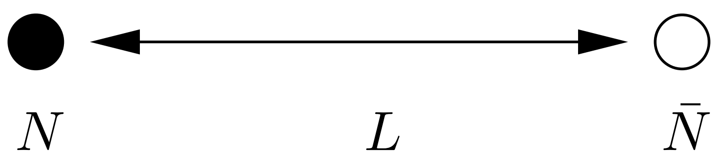
- Duality should hold for all coupling but in AdS/CFT the worldsheet description is problematic at weak coupling.
- LCWS approach directly sums planar diagrams on a lightcone worldsheet template.
- Once planar diagrams mapped to ws dynamics, nonperturbative questions can be addressed.
- Semi-classical string approximation not available for QCD but LCWS approach still applicable.

QCD vs. $\mathcal{N} = 4$ Super Yang-Mills

- QCD has a mass gap and confinement.
- QCD has scale dependent coupling: therefore no free parameters with massless quarks.
- $\mathcal{N} = 4$ super Yang-Mills is conformally invariant: no mass gap, no confinement, but a fixed coupling.
- For $\mathcal{N} = 4$, weak and strong coupling have meaning for all processes.
- For QCD, weak coupling only at high momentum and stronger coupling at low momentum arbitrarily strong coupling may never be reached.

2 $Q\bar{Q}$ System

A good test of confinement in gauge theory is its response to separated static color sources: a time-like rectangular Wilson Loop.



$$
\begin{aligned}
\langle W(L,T)\rangle \quad & \underset{T\to\infty}{\sim} \quad e^{-E(L)T} \\
& \underset{L,T\to\infty}{\sim} \quad e^{-T_0 LT} \qquad \text{for confinement} \\
& \underset{L,T\to\infty}{\sim} \quad e^{-cT/L} \qquad \text{conformal invariance}
\end{aligned}
$$

Some known properties of $Q\bar{Q}$ System:

- Flux lines connect quark to antiquark. We call the collection of all flux lines a flux tube.
- This flux tube has an excitation spectrum. At weak coupling, lowest state corresponds to Coulomb field with energy $E_G \sim -q^2/L$.
- In QED, vibrating flux lines just correspond to radiation of massless photons: spectrum is a continuum starting at E_G.
- Similar expectations in non-confining gauge theories, with radiation of massless gluons.
- Surprise in nonconfining gauge theories at $N = \infty$: a gap forms between the continuum and E_G.
  (Klebanov, Maldacena, Thorn (KMT)).
- For $\mathcal{N} = 4$ Supersymmetric gauge theories, AdS/CFT gives the strong coupling Flux Tube excitation spectrum

- Semi-classical quantization (small oscillations, Callan-Guijosa and Rey-Bak [4]) gives string like modes with discrete levels just above E_0:

$$E_{N_n} - E_0 \quad = \quad \sum N_n \omega_n$$

For large n, $\omega_n \sim (2\pi)^3(n+1)/\Gamma(1/4)^4 L$.

- Excitations of "string": $T_{eff} \sim 1/L^2$.

- Near threshold ($E = 0$) discrete levels accumulate (KMT)

$$\frac{E_{n+1}}{E_n} \sim e^{-\pi/2\sqrt{\lambda}}$$

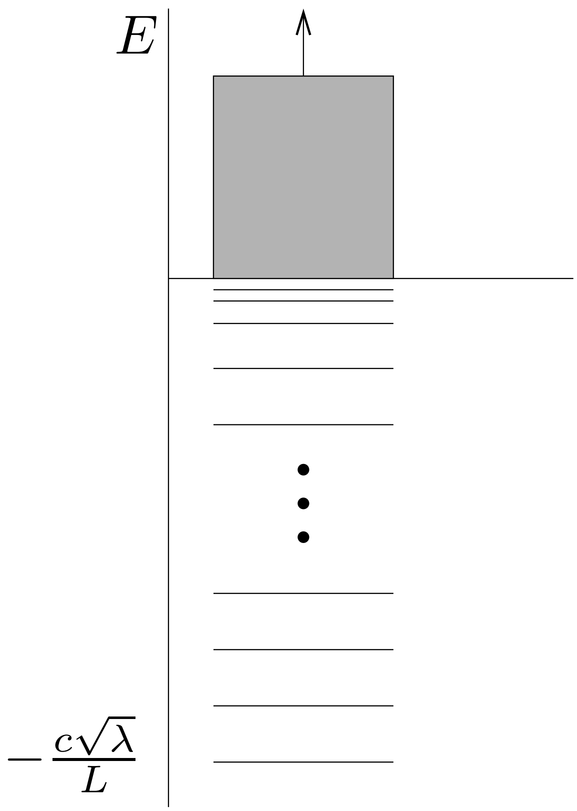
Shown by semiclassical motion of midpoint of string stretched almost to the horizon.

- $\mathcal{N} = 4$ **is teetering on the brink of quark confinement**:

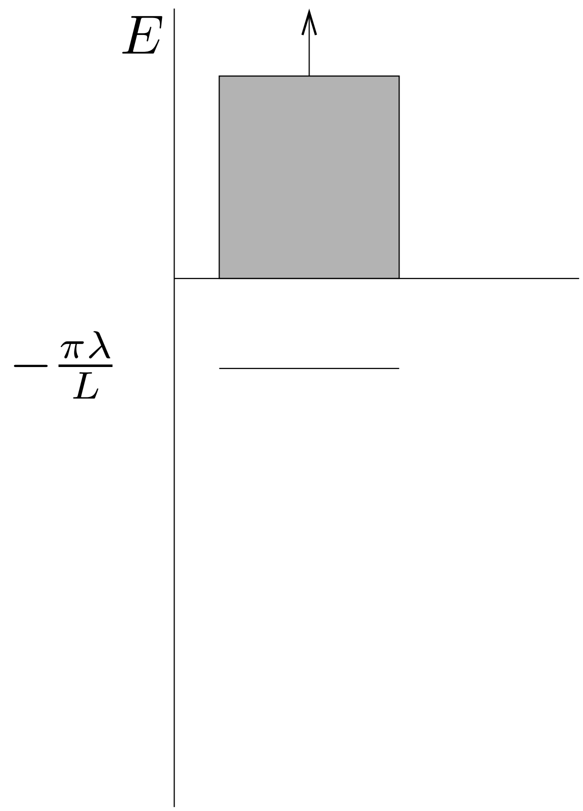
- Flux tube at fixed L is stringy but no mechanism prevents $T_{eff} \to 0$ as $L \to \infty$.

3 $\mathcal{N} = 4$: Strong vs. Weak

Energy level diagrams for $Q\bar{Q}$ Flux Tube at $N = \infty$ (KMT)



$\lambda \gg 1$



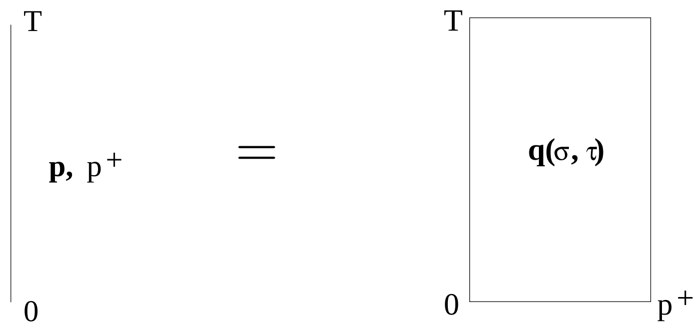
$\lambda \ll 1$

4 Discussion

- Transition from weak to strong is gradually stronger binding with a deepening gap that eventually supports excited discrete levels that peel off the continuum and move into the gap.
- This nice picture requires $N = \infty$. Otherwise the continuum goes all the way down to E_0.
- Ladder model of Erickson, Seminoff, Szabo, Zarembo [5] captures much of this physics, except at strong coupling there is only a single mode of small oscillations instead of an infinite number of stringy modes ω_n.
- Our goal: use lightcone worldsheet formalism to treat the complete sum of planar diagrams: going beyond ladders.

5 QFT Lightcone Worldsheet

Master Formula for Massless Propagator:

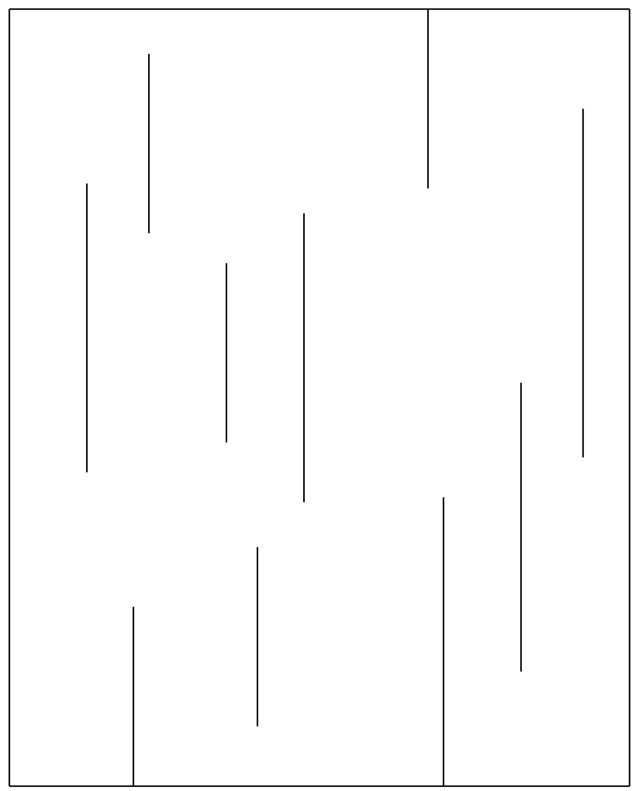


$$\exp\left\{-\frac{T}{2p^+}\boldsymbol{p}^2\right\} \quad = \quad \int_{\substack{\boldsymbol{q}(0,\tau)=0 \\ \boldsymbol{q}(p^+,\tau)=\boldsymbol{p}}} DcDbD\boldsymbol{q}\; e^{iS_0}$$

$$iS_0 \quad = \quad \int_0^T d\tau \int_0^{p^+} d\sigma \left(b'c' - \frac{1}{2}\boldsymbol{q}'^2\right)$$

- Dirichlet b.c.'s. Cf. string in momentum space
- Represents a field quantum as a composite of String Bits: Total $p^+ =$ (Number of bits)$\times m$.

The following is the worldsheet diagram describing a 7 loop 5 gluon diagram:



Beginnings and ends of vertical lines mark the cubic vertices of QFT.

Dirichlet boundary conditions on each vertical line

- The striking feature of this formalism is that the worldsheet dynamics is local (this has nothing to do with the locality of QFT)

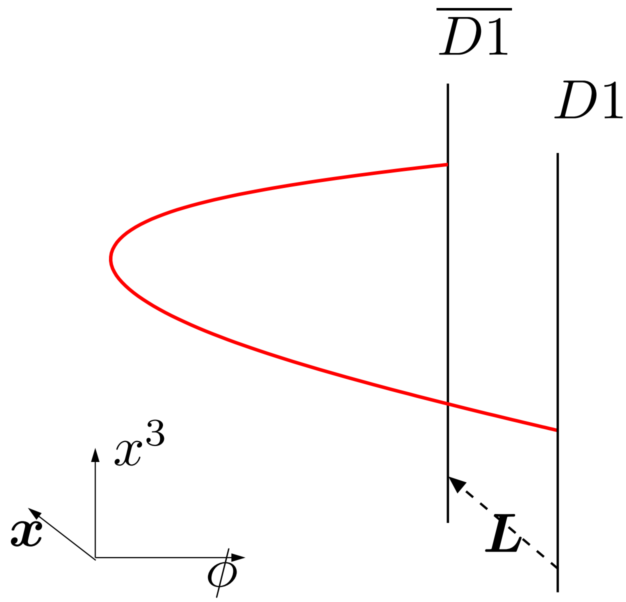
- As long as we consider unconstrained evoluion, local worldsheet dynamics is maintained.

- There are complications when applied to a flux tube with fixed ends: the Dirichlet boundary conditions must be modified to correspond to separated localized sources.

- Localizing the sources in x^- is especially problematic because conservation of p^+ would be lost.

To deal with the last complication we follow Rozowsky and Thorn [11]

- Localize the sources to 1-branes parallel to z-axis – this mainatins p^+ conservation.
- Particles (branions) living on the branes interact with those in the bulk.
- Put color rep N on one brane and rep $\bar{N}$ on the other
- Bulk gauge field couples to these branions: flux tube will form connecting the two branes.



For the AdS string which describes the strong coupling of the $\mathcal{N} = 4$ theory, this setup leads us to analyze the string spectrum of a string connecting two 1-branes. (Brower, Tan and Thorn [12])

6 $Q\bar{Q}$ Flux Tube on Lightcone Worldsheet

- The original lcws representation worked with Feynman diagrams in momentum space.
- In coordinate space the lightcone worldsheet representation is highly singular, requiring delta function measure factors.

This can be understood by noting how string tension enters the different representations of lightcone string dynamics:

Worldsheet Coordinates and Momenta of String:

$$x^\mu(\sigma,t), \qquad \mathcal{P}^\mu(\sigma,t)$$

Here $\mathcal{P}^\mu d\sigma = dp^\mu$ is the energy momentum carried by the element $d\sigma$ of string.

Lightcone: $\qquad x^+ \equiv \frac{1}{\sqrt{2}}(x^0+x^3)=t, \qquad \mathcal{P}^+=1$

$$S \;=\; \int dt \int_0^{p^+} d\sigma \frac{1}{2}\left(\dot{\boldsymbol{x}}^2 - T_0^2 \boldsymbol{x}'^2\right)$$

The limit $T_0 \to \infty$ forces $\boldsymbol{x}' \to 0$ everywhere in the bulk of the worldsheet.

In contrast, after T Duality: $(\boldsymbol{q}', \dot{\boldsymbol{q}}) = (\dot{\boldsymbol{x}}, -T_0^2 \boldsymbol{x}')$

$$\begin{aligned} iS \;&=\; -\int d\tau \int_0^{p^+} d\sigma \frac{1}{2}\left(\boldsymbol{q}'^2 + T_0^{-2} \dot{\boldsymbol{q}}^2\right) \\ &\to\; -\int d\tau \int_0^{p^+} d\sigma\, \frac{\boldsymbol{q}'^2}{2} \qquad \text{for } T_0 \to \infty \end{aligned}$$

7 Branion as N-D Lightcone WS

Gluon Propagator

$$\exp\left\{\frac{-\boldsymbol{p}^2T}{2p^+}\right\} \rightarrow \exp\left\{\frac{-(\boldsymbol{q}_1-\boldsymbol{q}_1)^2T}{2p^+}\right\}$$

Branion Propagator

$$\exp\left\{\frac{-\mu^2T}{2p^+}\right\} \rightarrow 1 \quad \text{for } \mu \rightarrow 0$$

Worldsheet Action

$$S = \int d\tau d\sigma \frac{1}{2}\boldsymbol{q}'^2$$

D-D boundary conditions:

$$\boldsymbol{q} = \boldsymbol{q}_0 + (\boldsymbol{q}_1 - \boldsymbol{q}_0)\sigma/p^+, \qquad S_{\rm DD} = (\boldsymbol{q}_1 - \boldsymbol{q}_0)^2/2p^+$$

Determinant prefactor: p^+/m

D-N conditions the coeff of σ must vanish, so

$$\boldsymbol{q} = \boldsymbol{q}_0, \qquad \boldsymbol{q}' = 0, \qquad S_{DN} = 0$$

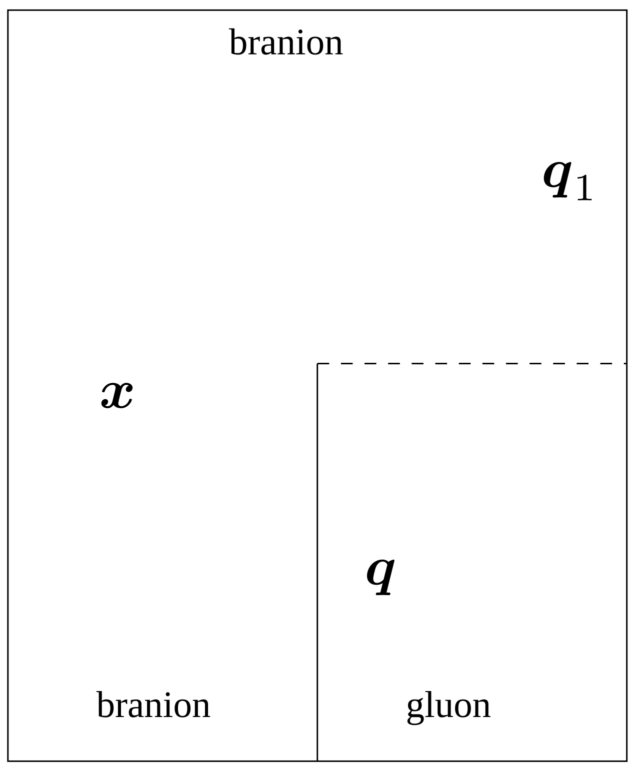
Determinant prefactor: 1.

Gluon-Branion Vertex on LCWS

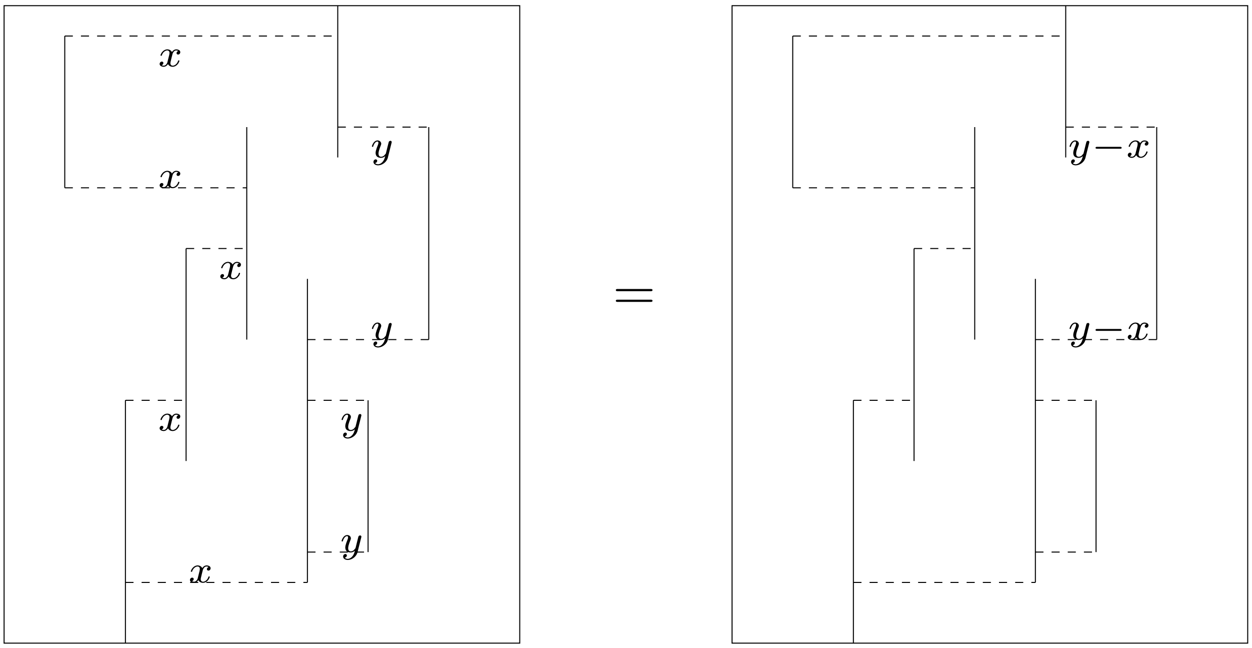
Coordinate space propagator

$$\int \frac{d^2p}{(2\pi)^2} e^{i(\boldsymbol{x}-\boldsymbol{y})\cdot\boldsymbol{p}} e^{-\boldsymbol{p}^2 T/2p^+}$$

Thus a factor $e^{i\boldsymbol{p}\cdot\boldsymbol{r}}$ accompanies a gluon branion-branion vertex, where the gluon momentum is $\boldsymbol{p} = \boldsymbol{q}_1 - \boldsymbol{q}$ and the brane is at transverse position $\boldsymbol{r}$.



8 Generic Planar Graph for $Q\bar{Q}$



- Each horizontal dashed line: a factor in the worldsheet path integral

$$e^{i\boldsymbol{x}\cdot\boldsymbol{p}} = e^{i\boldsymbol{x}\cdot\int d\sigma \boldsymbol{q}'}$$

- Still some worldsheet nonlocality when $y \neq x$.
- This needs to be fixed–I'm not sure how.

9 AdS String on LCWS

For a glimpse of our goal, look at AdS string in momentum space, starting with phase space PI [10]

$$S = \int dt \int_0^{p^+} d\sigma \left(\dot{\boldsymbol{x}} \cdot \boldsymbol{\mathcal{P}} - \frac{1}{2}\boldsymbol{\mathcal{P}}^2 - \frac{G^2(\phi)}{2}\boldsymbol{x}'^2 \right)$$

Change variables to $\boldsymbol{q}$ related to $\boldsymbol{\mathcal{P}}$ by $\boldsymbol{q}' = \boldsymbol{\mathcal{P}}$.

$$\begin{aligned} S &= \int dt \int_0^{p^+} d\sigma \left(\dot{\boldsymbol{x}} \cdot \boldsymbol{q}' - \frac{1}{2}\boldsymbol{q}'^2 - \frac{G^2(\phi)}{2}\boldsymbol{x}'^2 \right) \\ &= \int dt \int_0^{p^+} d\sigma \left(-\dot{\boldsymbol{x}}' \cdot \boldsymbol{q} - \frac{1}{2}\boldsymbol{q}'^2 - \frac{G^2(\phi)}{2}\boldsymbol{x}'^2 \right) \\ &\rightarrow \int dt \int_0^{p^+} d\sigma \left(\boldsymbol{x}' \cdot \dot{\boldsymbol{q}} - \frac{1}{2}\boldsymbol{q}'^2 - \frac{G^2(\phi)}{2}\boldsymbol{x}'^2 \right) \end{aligned}$$

where the surface term in the second line vanishes because we assume Dirichlet conditions on $\boldsymbol{x}$, and we drop the surface terms in the last line because they don't affect the dynamics.

Now integrate out $\boldsymbol{x}$, which determines

$$G^2(\phi)\boldsymbol{x}' = \dot{\boldsymbol{q}} + f(\tau)$$

But $\boldsymbol{q}$ was only defined up to a function of τ, so no generality is lost by setting $f = 0$: $\boldsymbol{x}' = \dot{\boldsymbol{q}} G^{-2}(\phi)$. Then the action interms of $\boldsymbol{q}$ becomes

$$S_q = \int dt \int_0^{p^+} d\sigma \left(-\frac{1}{2}\boldsymbol{q}'^2 + \frac{G^{-2}(\phi)}{2}\dot{\boldsymbol{q}}^2 \right)$$

Suppose that the Dirichlet conditions on $\boldsymbol{x}$ were $\boldsymbol{x}(p^+,\tau) - \boldsymbol{x}(0,\tau) = \mathbf{R}$. Then we have the non-local constraint

$$\boldsymbol{R} = \int_0^{p^+} d\sigma \frac{\dot{\boldsymbol{q}}}{G^2(\phi)}$$

Our challenge is to see a constraint like this coming out of the sum of planar diagrams represented on the Lightcone worldsheet.

10 Conclusions

- Fixed separated sources introduce complications in the momentum space lightcone worldsheet.

- At zero separation, changing ws boundary conditions from Dirichlet to Neumann works fairly simply.

- But then non-zero separation seems to require nonlocal elements in the ws dynamics.

- Field/String duality promises to shed light on many issues in theoretical physics.

- Our worldsheet description of quantum field theories can be developed for a wide class of matrix QFT's: it is nearly as generic as the $1/N$ expansion.

- In the next few years, the lightcone worldsheet version of field/string duality should yield insights on many issues from quark confinement to the foundations of string theory and quantum gravity.

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