Miami 2006, Fort Lauderdale, Florida

Higgs Boson Mass From Gauge-Higgs-Yukawa Unification

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Phys.Rev. D73, 066008 (2006)

The SM Higgs Boson Mass

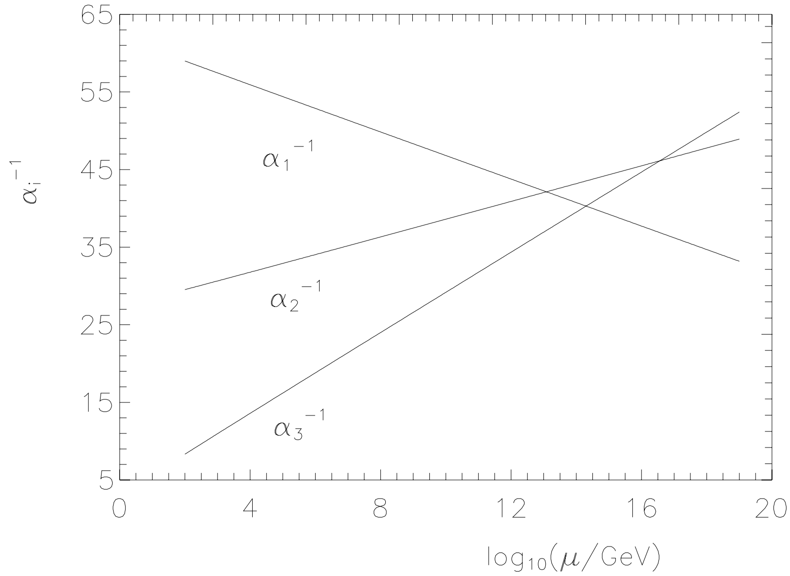
$$\mathbf{m_h = \sqrt{\lambda}\ v}$$

From the condition of perturbativity and triviality

$$\mathbf{0.8 < \sqrt{\lambda} < 1.1 \quad \Rightarrow \quad 130 < m_h < 180}$$

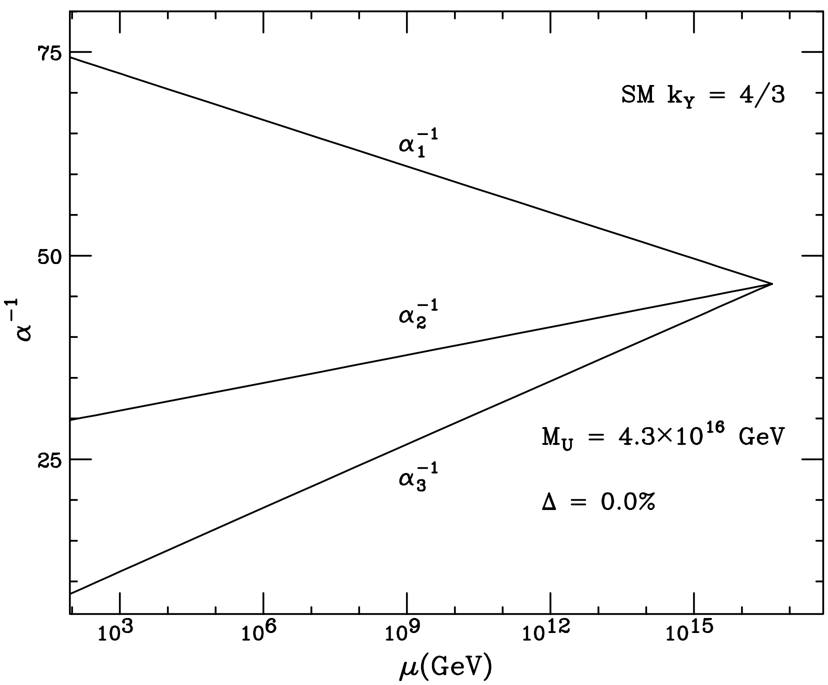
$$\frac{d}{dt}\,\lambda \;=\; \frac{\lambda}{16\pi^2}\left[12\lambda^2 - 12h_t^4 + \ldots\right]$$

Non-SUSY gauge coupling unification with canonical $U(1)_Y$ normalization



Is the canonical normalization ($k_Y = 5/3$) for $U(1)_Y$ unique?

V. Barger, J. Jiang, P. Langacker and T. Li, Nucl. Phys. B 726 (2005) 149, shown that in orbifold GUT there is possibility to have non-canonical normalization for $U(1)_Y$ hypercharge, which provides gauge coupling unification within the SM framework.



75
50
25
α^{-1}
SM k_Y = 4/3
α_1^{-1}
α_2^{-1}
α_3^{-1}
M_U = 4.3×10^{16} GeV
Δ = 0.0%
10^3
10^6
10^9
10^{12}
10^{15}
μ(GeV)

SU(7) orbifold model

The $\mathcal{N}=1$ SUSY in 7D corresponds to $\mathcal{N}=4$ SUSY in 4D. So, only the gauge supermultiplet can be introduced in the bulk. This multiplet can be decomposed under 4D $\mathcal{N}=1$ SUSY into a gauge vector multiplet V and three chiral multiplet Σ_1, Σ_2 and Σ_3 in the adjoint representation.

The bulk action in the Wess-Zumino gauge and in 4D N=1 SUSY notation contains:

$$\mathcal{S} = \int d^7x \, \mathrm{Tr}\left(\int d^2\theta \left(\frac{1}{kg^2}\Sigma_1[\Sigma_2, \Sigma_3]\right)\right) + h.c.$$

7D $\mathcal{N}=1$ SUSY $SU(7)$ gauge theory is compactified on the $T^2/Z_6 \times S^1/Z_2$ orbifold.

$$R_{\Gamma_T} = \text{diag}\,(+1,+1,+1,\omega^{n_1},\omega^{n_1},\omega^{n_1},\omega^{n_2})$$

$$R_{\Gamma_S} = \text{diag}\,(+1,+1,+1,+1,+1,-1,-1)$$

where n_1 and n_2 are positive integers, and $n_1 \neq n_2$.

$$\{SU(7)/R_{\Gamma_T}\} = SU(3)_C \times SU(3) \times U(1) \times U(1)'$$

$$\{SU(7)/R_{\Gamma_S}\} = SU(5) \times SU(2) \times U(1)$$

$$\mathbf{SU(7) \Rightarrow SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_\alpha \times U(1)_\beta}$$

$$\begin{pmatrix} (\mathbf{8},\mathbf{1})_{Q00} & (\mathbf{3},\bar{\mathbf{2}})_{\mathbf{Q12}} & (\mathbf{3},\mathbf{1})_{Q13} & (\mathbf{3},\mathbf{1})_{Q14} \\ (\bar{\mathbf{3}},\mathbf{2})_{Q21} & (\mathbf{1},\mathbf{3})_{Q00} & (\mathbf{1},\mathbf{2})_{\mathbf{Q23}} & (\mathbf{1},\mathbf{2})_{Q24} \\ (\bar{\mathbf{3}},\mathbf{1})_{\mathbf{Q31}} & (\mathbf{1},\bar{\mathbf{2}})_{\mathbf{Q32}} & (\mathbf{1},\mathbf{1})_{Q00} & (\mathbf{1},\mathbf{1})_{Q34} \\ (\bar{\mathbf{3}},\mathbf{1})_{Q41} & (\mathbf{1},\bar{\mathbf{2}})_{Q42} & (\mathbf{1},\mathbf{1})_{Q43} & (\mathbf{1},\mathbf{1})_{Q00} \end{pmatrix}$$

Unification of Gauge and Top Quark Yukawa Couplings

$$R_{\Gamma_T} = \text{diag}\left(+1,+1,+1,\omega^5,\omega^5,\omega^5,\omega^2\right)$$

$$R_{\Gamma_S} = \text{diag}\left(+1,+1,+1,+1,+1,-1,-1\right)$$

$$\begin{aligned}\mathbf{T}_{\mathbf{U(1)_Y}} &\equiv \frac{\mathbf{1}}{\mathbf{6}}\text{diag}\left(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{0},\mathbf{0},-\mathbf{3},\mathbf{0}\right)\\ &+ \frac{\sqrt{14}}{\mathbf{42}}\text{diag}\left(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1},-\mathbf{6}\right),\end{aligned}$$

$$\begin{aligned}\mathbf{T}_{\mathbf{U(1)_\alpha}} &\equiv -\frac{\sqrt{14}}{\mathbf{2}}\ \text{diag}\left(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{0},\mathbf{0},-\mathbf{3},\mathbf{0}\right)\\ &+ \text{diag}\left(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1},-\mathbf{6}\right)\end{aligned}$$

$$\mathbf{T}_{\mathbf{U(1)_\beta}} \equiv \text{diag}\left(\mathbf{1},\mathbf{1},\mathbf{1},-\mathbf{2},-\mathbf{2},\mathbf{1},\mathbf{0}\right)$$

$\text{tr}[T^2_{U(1)_Y}] = 2/3$. From $k_Y g_Y^2 = g_2^2 = g_3^2$ condition we are getting $\mathbf{k_Y = 4/3}$.

Zero modes from the chiral multiplets

$$\mathbf{\Sigma_1 \rightarrow Q_3 : \left(3, 2, \frac{1}{6}, 3, -\frac{\sqrt{14}}{2}\right)}$$

$$\mathbf{\Sigma_2 \rightarrow H_u : \left(1, 2, \frac{1}{2}, -3, -\frac{3\sqrt{14}}{2}\right)}$$
$$\mathbf{+H_d : \left(1, 2, -\frac{1}{2}, 3, \frac{3\sqrt{14}}{2}\right)}$$

$$\mathbf{\Sigma_3 \rightarrow t^c : \left(\bar{3}, 1, -\frac{2}{3}, 0, 2\sqrt{14}\right)}$$

from the trilinear term in the 7D bulk action

$$\int d^7x \left[\int d^2\theta \ g_7 Q_3 t^c H_u + h.c.\right],$$

$$g_1 = g_2 = g_3 = y_t = g_7/\sqrt{V},$$

1) The brain localized gauge and Yukawa interaction can be negligible.

2) The zero modes are not localized at different points on the orbifold

3) the four dimensional fields are not largely mixed with other brane localized fields.

Higgs Sector

$$H \equiv -\cos\beta\, i\sigma_2\, H_d^* + \sin\beta\, H_u$$

$$h_t = y_t \sin\beta$$

The quartic Higgs coupling is determined at M_{GUT} by the supersymmetric D-term

$$\lambda = \frac{\frac{3}{4}g_1^2(M_{\mathrm{GUT}}) + g_2^2(M_{\mathrm{GUT}})}{4}\cos^2 2\beta$$

$$m_h = \sqrt{\lambda}v$$

Numerical results from Gauge and Top Yukawa Unification

Compactification scale, SUSY and gauge symmetry breaking scale $\mathbf{SU(3)_C \times SU(2)_L \times U(1)_Y}$ is the same and equal M_{GUT}.
Below M_{GUT} scale we have usual the SM particle content with:

$$\mathbf{g_1 = g_2 = g_3 = y_t}, \qquad \mathbf{h_t = y_t} \sin\beta$$

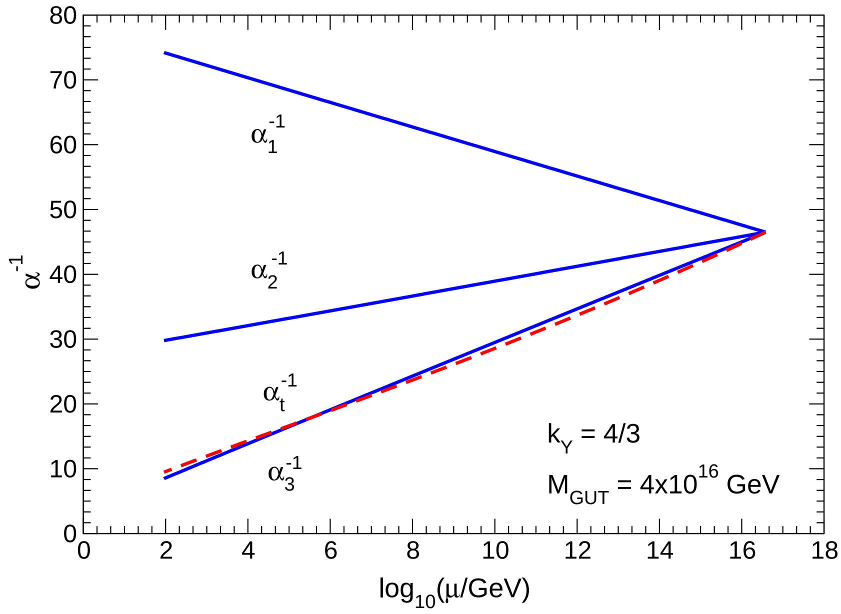
Input parameters:

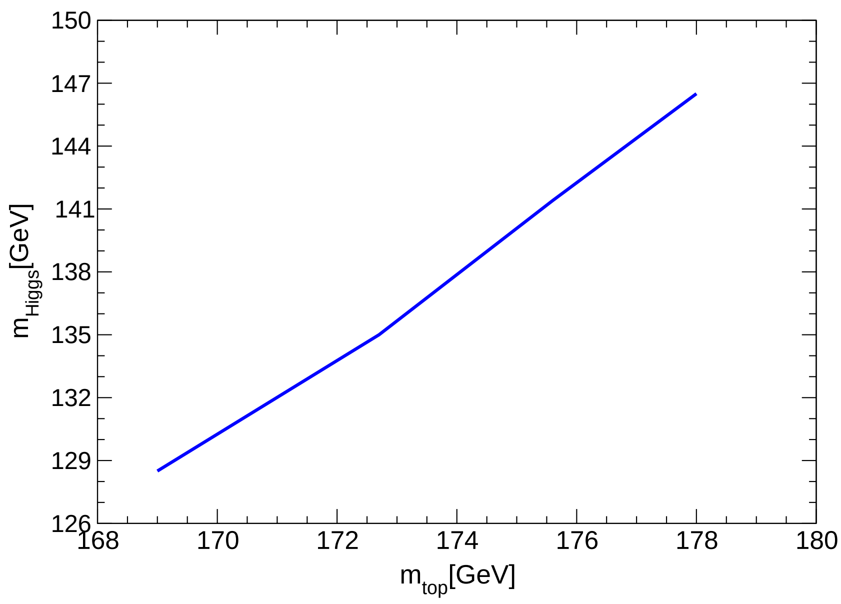
$$\begin{aligned} \alpha_{EM} &= \mathbf{127.92 \pm 0.02}, \qquad \mathbf{m_t = 172.7 GeV} \\ \sin^2\theta_w &= \mathbf{0.2311 \pm 0.0001}, \end{aligned}$$

Prediction:

$$\alpha_3(\mathbf{M_Z}) = \mathbf{0.118}, \qquad \mathbf{m_h = 135 GeV}$$

Unification of Gauge and top Yukawa couplings





Unification of Gauge and Bottom Quark Yukawa Couplings

$$R_{\Gamma_T} = \text{diag}\left(+1,+1,+1,\omega^5,\omega^5,\omega^5,\omega^2\right)$$

$$R_{\Gamma_S} = \text{diag}\left(+1,+1,+1,+1,+1,-1,-1\right)$$

$$\begin{aligned}\mathbf{T}_{\mathbf{U(1)_Y}} &\equiv -\frac{\mathbf{1}}{\mathbf{6}}\text{diag}\left(\mathbf{0},\mathbf{0},\mathbf{0},\mathbf{1},\mathbf{1},-\mathbf{2},\mathbf{0}\right)\\ &+ \frac{\sqrt{21}}{\mathbf{42}}\text{diag}\left(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1},-\mathbf{6}\right)\end{aligned}$$

$$\begin{aligned}\mathbf{T}_{\mathbf{U(1)_\alpha}} &\equiv \sqrt{21}\ \text{diag}\left(0,0,0,1,1,-2,0\right)\\ &+ \text{diag}\left(1,1,1,1,1,1,-6\right)\end{aligned}$$

$$\mathbf{T}_{\mathbf{U(1)_\beta}} \equiv \text{diag}\left(\mathbf{1},\mathbf{1},\mathbf{1},-\mathbf{1},-\mathbf{1},-\mathbf{1},\mathbf{0}\right)$$

$\text{tr}[T^2_{U(1)_Y}] = 2/3$. From $k_Y g_Y^2 = g_2^2 = g_3^2$ condition we are getting $\mathbf{k_Y = 4/3}$.

Zero modes from the chiral multiplets

$$\mathbf{\Sigma_1 \rightarrow Q_3 : \left(3, 2, \frac{1}{6}, 2, -\sqrt{21}\right)}$$

$$\mathbf{\Sigma_2 \rightarrow H_u : \left(1, 2, \frac{1}{2}, 0, -3\sqrt{14}\right)}$$
$$\mathbf{+H_d : \left(1, 2, -\frac{1}{2}, 0, 3\sqrt{14}\right)}$$

$$\mathbf{\Sigma_3 \rightarrow b^c : \left(\bar{3}, 1, \frac{1}{3}, -2, -2\sqrt{21}\right)}$$

from the trilinear term in the 7D bulk action

$$\int d^7x \left[\int d^2\theta \;\, g_7 Q_3 b^c H_d + h.c.\right]$$

$$g_1 = g_2 = g_3 = y_b = g_7/\sqrt{V}$$

$$h_b = y_b \cos\beta = g_7/\sqrt{V}$$

Numerical results from Gauge and Top Yukawa Unification

Compactification scale, SUSY and gauge symmetry breaking scale $\mathbf{SU(3)_C \times SU(2)_L \times U(1)_Y}$ is the same and equal M_{GUT}.
Below M_{GUT} scale we have usual the SM particle content with:

$$\mathbf{g_1 = g_2 = g_3 = y_b}, \qquad \mathbf{h_b = y_b} \cos \beta$$

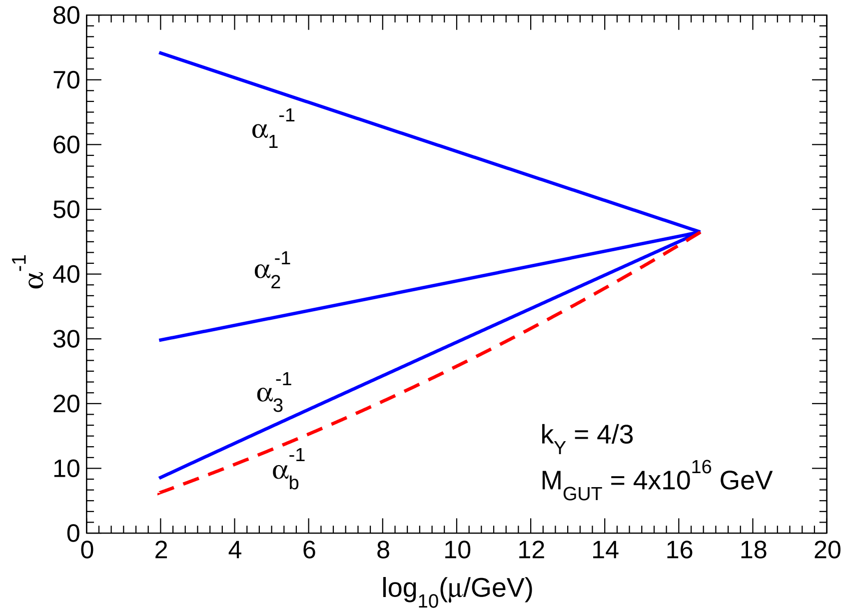
Input parameters:

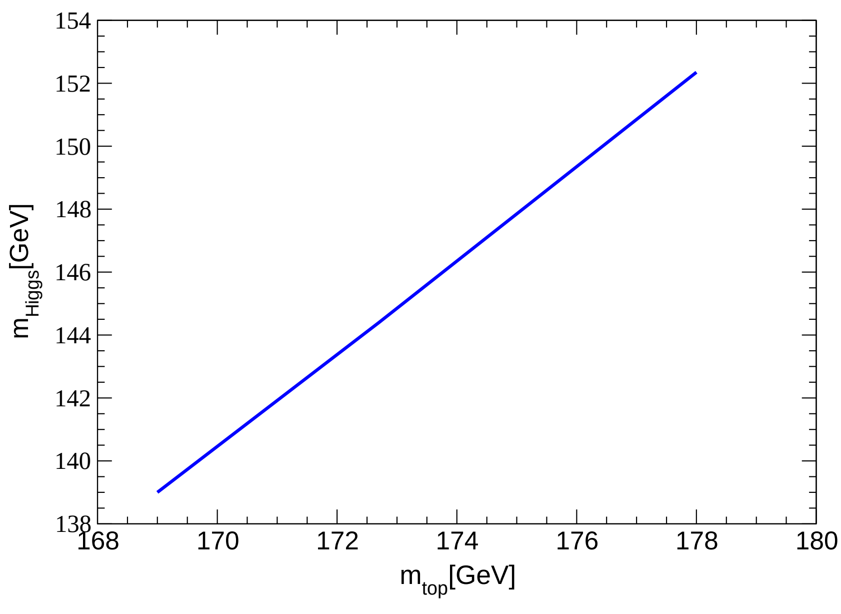
$$\begin{aligned} \alpha_{EM} &= \mathbf{127.92 \pm 0.02}, \qquad \mathbf{m_t = 172.9 GeV} \\ \sin^2 \theta_w &= \mathbf{0.2311 \pm 0.0001}, \end{aligned}$$

Prediction:

$$\alpha_{\mathbf{3}}(\mathbf{M_Z}) = \mathbf{0.118}, \qquad \mathbf{m_h = 144 GeV}$$

Unification of Gauge and bottom Yukawa couplings





Conclusion

We have considered **SU(7)** orbifold GUTs with $\mathcal{N} = 1$ supersymmetry in 7D. In this model the mass of the SM Higgs boson can be reliably predicted. For instance, using the measured value for top quark mass $m_t = 172.9$ GeV we predicate

- For gauge-top quark Yukawa coupling unification case $\mathbf{m_h = 135}$ GeV.

- For gauge-bottom quark Yukawa coupling unification case $\mathbf{m_h = 144}$ GeV

Getting no-canonical normalization for $U(1)_Y$ hypercharge **(**$k_Y = 4/3$**)**from SU(7) orbifold GUT we have perfecte two loop gauge coupling unification for non-SUSY theory.