Thank you for the invitation.

TURNAROUND

IN

CYCLIC COSMOLOGY

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OUTLINE

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SUMMARY.

Reference:

L.Baum and PF. `hep-th/0610213`

1. Historical context.

One of the oldest questions in theoretical cosmology is whether an infinitely oscillatory universe which avoids an initial singularity can be consistently constructed. As realized by Friedmann and especially by Tolman (also LeMaitre, Einstein, De Sitter ....) one principal obstacle is the second law of thermodynamics which dictates that the entropy increases from cycle to cycle. If the cycles thereby become longer, extrapolation into the past will lead back to an initial singularity again, thus removing the motivation to consider an oscillatory universe in the first place. This led to the abandonment of the oscillatory universe by the majority of workers.

Nevertheless, an oscillatory universe is an attractive alternative to the Big Bang. One new ingredient in the cosmic make-up is the dark energy discovered only in 1998 and so it natural to ask whether this can avoid the difficulties with entropy which have dogged previous attempts.

Some work has been started to exploit the dark energy in allowing cyclicity possibly without apparently the need for inflation in Steinhardt *et al* Another new ingredient is the use of branes and a fourth spatial dimension as in Randall *et al*, Binetruy *et al* which have examined the consequences for cosmology. The Big Rip and replacement of dark energy by modified gravity have been explored in PHF and Takahashi.

If the dark energy has a super-negative equation of state, $\omega_\Lambda = p_\Lambda / \rho_\Lambda < -1$, it leads to a Big Rip (R. Caldwell) at a finite time where there exist extraordinary conditions with regard to density and causality as one approaches the Big Rip. In the present article we explore whether these exceptional physical conditions can assist in providing an infinitely-cyclic cosmology.

We shall consider the situation where if, as we approach the Big Rip, the expansion stops due to the brane contribution just short of the Big Rip and there is a turnaround at $t = t_T$ when the scale factor is deflated to a very tiny fraction (f) of itself and only one causal patch is retained, while the other $1/f^3$ patches contract independently into separate universes. The turnaround takes place an extremely short time before the Big Rip would have occurred, at a time when the universe is fractionated into many independent causal patches, see *e.g.* PHF and Takahashi (2004).

We discuss the contraction phase which occurs with a very much smaller universe than in the expansion phase and with almost vanishing entropy because it is assumed empty of dust, matter and black holes all of which were jettisoned at turnaround. A bounce at $t = \tau$ takes place a short time before a would-be Big Bang. Then, immediately after the bounce, entropy is injected by inflation (Guth) where the scale factor is enhanced by large factor and hence so is entropy. Inflation can thus be a part of the present scenario which is one distinction from the work of Steinhardt *et al.*

For cyclicity of the entropy, $S(t) = S(t+\tau)$ to be consistent with thermodynamics it is necessary that the deflationary decrease by f^3 compensate the entire entropy increase acquired during contraction and expansion including the huge increase during inflation.

A possible shortcoming of the proposal could have been the persistence of spacetime singularities in cyclic cosmologies (Borde, Guth and Vilenkin, 2003) but to our understanding for the truly cyclic universe which we here outline this problem is avoided, provided a simple constraint on the time average of the Hubble parameter is respected.

This work is presented because our discussion seems to give a plausible realization of the infinitely oscillatory universe originally saught by cosmologists on the 1920s and 1930s ignorant of dark energy

(see, however, the discussion after Eq.(172.6) of R.C. Tolman in Relativity, Thermodynamics and Cosmology. Oxford University Press (1934))

and one whose minor shortcomings can hopefully be evolved by others into a convincing scenario.

2. Expansion phase.

Let the period of the Universe be designated by τ and the bounce take place at $t = 0$ and turnaround at $t = t_T$. Thus the expansion phase is for times $0 < t < t_T$ and the contraction phase corresponds to times $t_T < t < \tau$. We employ the following Friedmann equation for the *expansion* period $0 < t < t_T$:

$$\left(\frac{\dot{a}(t)}{a(t)}\right)^2 = \frac{8\pi G}{3}\left[\left(\frac{(\rho_\Lambda)_0}{a(t)^{3(\omega_\Lambda+1)}} + \frac{(\rho_m)_0}{a(t)^3} + \frac{(\rho_r)_0}{a(t)^4}\right) - \frac{\rho_{total}(t)^2}{\rho_c}\right] \tag{1}$$

where the scale factor is normalized to $a(t_0) = 1$ at the present time $t = t_0 \simeq 14Gy$.

To explain the notation, $(\rho_i)_0$ denotes the value of the density ρ_i at time $t = t_0$. The first two terms are the dark energy and total matter (dark plus luminous) satisfying

$$\Omega_\Lambda = \frac{8\pi G(\rho_\Lambda)_0}{3H_0^2} = 0.72 \tag{2}$$

and

$$\Omega_m = \frac{8\pi G(\rho_m)_0}{3H_0^2} = 0.28 \tag{3}$$

where $H_0 = \dot{a}(t_0)/a(t_0)$. The third term in the Friedmann equation is the radiation density which is now $\Omega_r = 1.3 \times 10^{-4}$.

The final term $\sim \rho_{total}(t)^2$ is derivable from a brane set-up; we use a negative sign arising from negative brane tension (a negative sign can arise also from a second timelike dimension but that gives difficulties with closed timelike paths). $\rho_{total} = \Sigma_{i=\Lambda,m,r}\rho_i$. As the turnaround is approached, the only significant terms in Eq.(1) are the first (where $\omega_\Lambda < -1$) and the last.

As the bounce is approached, the only important terms in Eq.(1) are the third and the last. (We shall later argue that the second term must be absent during contraction.) In particular, the final term of Eq. (1), $\sim \rho_{total}(t)^2$, arising from the brane set up is insignificant for almost the entire cycle but becomes dominant as one approaches $t \rightarrow t_T$ for the turnaround and again for $t \rightarrow \tau$ approaching the bounce.

3. Turnaround.

Let us assume for algebraic simplicity $\omega_\Lambda = -4/3 =$ constant. This value is already almost excluded by WMAP3 but to begin we are aiming only at consistency of infinite cyclicity. More realistic values may be discussed elsewhere. The approach to the Big Rip will follow that discussed in PHF+TT *q.v.*. With the value $\omega_\Lambda = -4/3$ we learn therefrom that the time to the Big Rip is $(t_{rip} - t_0) = 11\text{Gy}(-\omega_\Lambda - 1)^{-1} = 33\text{Gy}$ which is, within 10^{-27} second, when turnaround occurs at $t = t_T$. So if we adopt $t_0 = 14Gy$ then $t_T = t_0 + (t_{rip} - t_0) = (14 + 33)Gy = 47Gy$.

From the analysis in PHF+TT the time when a system becomes gravitationally unbound corresponds approximately to the time when the growing dark energy density matches the mean density of the bound system. For a "typical" object like the Earth (or a hydrogen atom where the mean density happens to be about the density of water $\rho_{H_2O} = 1g/cm^3$ since $10^{-24}g/(10^{-8}cm)^3 = 1g/cm^3$) water's density ρ_{H_2O} is an unlikely but practical unit for cosmic density in the oscillatory universe.

With this in mind, for the simple case of $\omega = -4/3$ we see from the Friedmann equation that the dark energy density grows proportional to the scale factor $\rho_\Lambda(t) \propto a(t)$ and so given that the dark energy at present is $\rho_\Lambda \sim 10^{-29} g/cm^3$ it follows that $\rho_\Lambda(t_{H_2O}) = \rho_{H_2O}$ when $a(t_{H_2O}) \sim 10^{29}$. We can estimate the time t_{H_2O} by taking on the RHS of the Friedmann equation only dark energy $\left(\frac{\dot{a}}{a}\right)^2 = H_0^2 \Omega_\Lambda a^{-\beta}$ with $\beta = 3(1+\omega)$. When we specialize to $\omega = -4/3$ as illustration and require as before $\rho_\Lambda(t_{H_2O}) = \rho_{H_2O}$ then $a(t_{H_2O}) = 10^{29}$ and it follows that

$$\frac{a(t_{H_2O})}{(a(t_0) = 1)} = \left(\frac{(t_{rip} - t_0)}{(t_{rip} - t_{H_2O})} \right)^2 \tag{4}$$

so that $(t_{rip} - t_{H_2O}) = 33Gy \times 10^{-14.5} \simeq 10^{3.5} s \sim 1$ hour. [The value is sensitive to ω]

For $\omega = -4/3$, it will be useful to consider a more general critical density $\rho_c = \eta\rho_{H_2O}$, since there is nothing special about ρ_{H_2O} and to compute the time $(t_{rip} - t_\eta)$ such that $\rho_\Lambda(t_\eta) = \rho_c = \eta\rho_{H_2O}$. We then find, using $a(t_\eta) = 10^{29}\eta$, that $(t_{rip} - t_\eta) = (t_{rip} - t_0)10^{-14.5}\eta^{-1} \simeq \eta^{-1}$hours is the required result.

To discuss the turnaround analytically we keep only the first and last terms, the only significant ones, on the RHS of the Friedmann equation which becomes for the special case $\omega = 4/3$

$$\left(\frac{\dot{a}}{a}\right)^2 = \alpha_1 a - \alpha_2 a^2 \tag{5}$$

in which

$$\alpha_1 = \frac{8\pi G}{3}(\rho_\Lambda)_0 \qquad \alpha_2 = \frac{8\pi G}{3}\frac{(\rho_\Lambda)_0^2}{\rho_c} \tag{6}$$

Writing $a = z^2$ and $z = (\alpha_1/\alpha_2)^{1/2} sin\theta$ gives

$$dt = \frac{2\sqrt{\alpha_2}}{\alpha_1}\frac{d\theta}{sin^2\theta} = \frac{2\sqrt{\alpha_2}}{\alpha_1}d(-cot\theta) \tag{7}$$

Integration then gives for the scale factor

$$a(t) = \left(\frac{\alpha_1}{\alpha_2}\right) sin^2\theta = \frac{\rho_c}{(\rho_\Lambda)_0}\left[\frac{1}{1+\left(\frac{t_T - t}{C}\right)^2}\right] \quad (8)$$

where $C = -(3/2\pi G\rho_c)^{1/2}$. At turnaround $t = t_T$, $a(t_T) = [\rho_C/(\rho_\lambda)_0] = (a(t))_{max}$. At the present time $t = t_0$, $a(t_0) = 1$ and $sin^2\theta_0 = [(\rho_\Lambda)_0/\rho_C] \ll 1$, increasing during subsequent expansion to $\theta_T = \pi/4$.

A key ingredient in our cyclic model is that at turnaround $t = t_T$ (mod τ) our universe deflates dramatically with effective scale factor $a(t_T)$ shrinking before contraction to $\hat{a}(t_T) = fa(t_T)$ where $f < 10^{-28}$. This jettisoning of almost all, a fraction $(1 - f)$, of the accumulated entropy may be permitted by the exceptional causal structure of the universe. We shall see later that the parameter η at turnaround could be $\eta \sim 10^{31}$ or even larger which implies the dark energy density at turnaround of $\rho_\Lambda(t_T) > 10^{31}\rho_{H_2O}$ (Planckian density of $\rho_\Lambda \sim 10^{104}\rho_{H_2O}$ can be avoided). By the time the dark energy density reaches such values, according to the Big Rip analysis of PHF+TT even the smallest known bound systems of particles have become unbound and the constiutents causally disconnected. Possibly smaller unknown bound systems have equally become unbound and acausal.

According to this, at $t = t_T$ the universe has already fragmented into an astronomical number $(1/f^3)$ of causal patches, each of which independently contracts as a separate universe leading to an infinite multiverse. The entropy at $t = t_T$ is thus divided between these new contracting universes and our universe retains only the infinitesimal fraction f^3. Since our eternal universe has cycled an infinite number of times, the number of parallel universes is infinite.

4. Contraction phase.

The contraction phase for our universe occurs for the period $t_T < t < \tau$ (mod τ). The scale factor for the contraction phase will be denoted by $\hat{a}(t)$ while we use always the same linear time t subject to the periodicity $t + \tau \equiv t$. At the turnaround we retain a fraction f^3 of the entropy with $\hat{a}(t_T) = f a(t_T)$ and for the contraction phase the Friedmann equation is:

$$\left(\frac{\dot{\hat{a}}(t)}{\hat{a}(t)}\right)^2 = \frac{8\pi G}{3}\left[\left(\frac{(\hat{\rho}_\Lambda)_0}{\hat{a}(t)^{3(\omega_\Lambda+1)}} + \frac{(\hat{\rho}_r)_0}{\hat{a}(t)^4}\right) - \frac{\hat{\rho}_{total}(t)^2}{\rho_c}\right] \tag{9}$$

where we defined

$$\hat{\rho}_i(t) = \frac{(\rho_i)_0 f^{3(\omega_i+1)}}{\hat{a}(t)^{3(\omega_i+1)}} = \frac{(\hat{\rho}_i)_0}{\hat{a}(t)^{3(\omega_i+1)}} \tag{10}$$

In contrast to expansion we have set $\hat{\rho}_m = 0$ because our hypothesis is that the causal patch retained contains only dark energy and radiation but no matter (no black holes). This is necessary because during a contracting phase dust or matter would clump, more readily than during expansion, and interfere with cyclicity.

Perhaps more importantly, presence of dust or matter would require that our universe go in reverse through several phase transitions (recombination, QCD and electroweak to name a few) which would violate the second law of thermodynamics and be statistically impossible.

We thus require that

our universe comes back empty!

(like a milk bottle in the old days)

The contraction of our universe will proceed from one of the $1/f^3$ causal patches following the truncated Friedmann equation until the radiation term balances the brane tension term at the bounce.

5. Bounce

As an estimated time $t = \tau$ for the bounce, the contraction scale is given, using $\rho_c = \eta\rho_{H_2O}$, from Eq. (1), and using the certainty that $(\rho_a)_0 < (\rho_r)_0$ as

$$a(\tau)^4 = \left(\frac{(\rho_a)_0}{\eta\rho_{H_2O}}\right) = \left(\frac{10^{-33}}{\eta}\right) \qquad (11)$$

Now the bounce at $t = \tau$ must be before the electroweak transition at $t_{EW} = 10^{-10}s$ when $a(t_{EW}) = 10^{-15}$ and after the Planck time where $a(t_{Planck}) \sim 10^{-31}$ (Recall $a \propto T^{-1}$).

We may take as illustrative cases:

- $T_B = 10^{17}GeV, a(t_B) = 10^{-30}, \eta = 10^{87}$
- $T_B = 10^{10}GeV, a(t_B) = 10^{-23}, \eta = 10^{59}$
- $T_B = 10^{3}GeV, a(t_B) = 10^{-16}, \eta = 10^{31}$

Immediately after the bounce there is conventional inflation with enhancement $\mathrm{E} = a(\tau + \delta)/\hat{a}(\tau)$ and successful inflation requires $E > 10^{28}$. Consistency requires therefore $f < E^{-1}$ to allow for the entropy accrued during normal expansion after inflation and contraction. The fraction of entropy jettisoned from our universe at deflation at the turnaround is thus extremely close to one, being less than one and more than $(1 - 10^{-28})^3$.

6. Entropy.

Consider first the present epoch $t = t_0$. The contributions of the radiation to the entropy density s follows the relation

$$s = \frac{2\pi^2}{45} g_* T^3 \tag{12}$$

First consider only photons with $g_* = 2$. The present CMB temperature is $T = 2.73K \equiv 0.235meV \sim 1.191(mm)^{-1}$. Substitution in Eq.(12) gives a present radiation entropy density $s_\gamma(t_0) = 1.48(mm)^{-3}$. Using a volume estimate $V = (4\pi/3)R^3$ with $R = 10Gly \simeq 10^{29}mm$ gives a total radiation entropy $S_\gamma \sim 6.3 \times 10^{87}$. Including neutrinos increase g_* in Eq.(12) from $g_* = 2$ to $g_* = 3.36 = 2 + 6 \times (7/8) \times (4/11)^{4/3}$. This increases $S_\gamma = 6.3 \times 10^{87}$ to $S_{\gamma+\nu} \sim \times 10^{88}$.

The total entropy is interpretable as $exp(10^{88})$ degrees of freedom, or in information theory to a number I of qubits where $2^I = e^S$ so that $I = S/(ln2 = 0.693) \sim 10^{88}$. This is well below the holographic bound which is dictated by the area in terms of Planck units $10^{-64}mm^2$ which gives $S_{holog}(t_0) = 4\pi(10^{29}mm)^2/(10^{-32}mm)^2 \sim 10^{123}$ about 10^{35} times bigger. Some of this difference may come from supermassive black holes.

The entropy contribution from the baryons is smaller than S_γ by some ten orders of magnitude, so like that of the dark matter, is negligible. What is the entropy of the dark energy? If it is perfectly homogeneous and non-interacting it has zero for both temperature and entropy. Another viewpoint, at least for a pure cosmological constant, is that one number Λ cannot contain entropy. Finally, the 4th term in Eq.(1) corresponding to the brane term is neglible, as we have already estimated.

The conclusion is that $S_{total}(t_0) \sim 10^{88}$. Now consider the entropy at turnaround $t = t_T$ (mod τ). We have estimated that $a(t_T) = 10^{29}\eta$. The temperature T_γ of the radiation scales as $T_\gamma \propto a(t)^{-1}$ so using the entropy density of Eq.(12) a comoving 3-volume $\propto a(t)^3$ will contain the same total radiation entropy $S_\gamma(t_T) = S_\gamma(t_0)$ as at present; this is simply the usual adiabatic expansion.

The expansion from $t = 0$ (mod τ) to t_T (mod τ) is, of course, not purely adiabatic because irreversible processes take place. The first is inflation which increases entropy by $> 10^{84}$. There are phase transitions such as the electroweak transition at $t_{ew} \sim 100ps$, the QCD phase transition at $t_{QCD} \sim 100\mu s$, and recombination at $t_{rec} \sim 10^{13}s$. Further irreversible processes occur during during stellar evolution. Although the expansion of the radiation, the dominant contributor to the entropy, is adiabatic, the entropy of the matter inevitably increases with time in accord with the second law of thermodynamics.

In our model, the entropy of the matter increases between $t = 0 \ (\bmod \ \tau)$ and $t_T = 47Gy$. Setting the entropy of the dark energy to zero and the radiation as adiabatic, the matter part represented by ρ_m will cause the entropy to rise from $S(t = 0)$ to $S(t_T) = S(t = 0) + \Delta S$ where ΔS causes the contradiction plaguing the oscillatory universe in the 1920s and 1930s. The key point is that in order for entropy to be cyclic, the entropy which was enhanced by a huge factor $E^3 > 10^{84}$ at inflation must be reduced even more dramatically at some point during the cycle so that $S(t) = S(t + \tau)$ becomes possible. Since it increases during both expansion and contraction, the only logical possibility is a dramatic decrease at turnaround as accomplished by our hypothesis of one causal patch retention .

The second law or thermodynamics continues to obtain for other causally disconnected regions, each with practically vanishing entropy at turnaround, but these are permanently removed from our universe contracting instead into separate universes.

Next, we look at entropy for contraction $t_T < t < \tau$ (mod τ). According to statistical mechanics one expects the entropy to increase here also, although because it is much smaller the increase must be correspondingly much smaller than during expansion and as we are assuming the universe during contraction is empty of dust until the bounce its entropy is, in any case, vanishingly small for the contraction era.

Finally, there is the issue of entropy at bounce $t = \tau \pmod{\tau}$. Immediately after the bounce inflation increases entropy by $> 10^{84}$ so cyclicity $S(t) = S(t+\tau)$ is possible providing the entropy loss at turnaround compensates the gain during inflation as well as the other entropy acquisitions. We find the counterpoise of inflation at the bounce and deflation at turnaround an appealing aspect.

It is worth a mention that there exists an argument from the conventional Friedmann equation (see Misner, Thorne and Wheeler book) that when the RHS terms all have equation of state $\omega \geq -1$ then $\ddot{a} < 0$ which disallows a bounce after which the universe starts expanding again. In the present model the brane term contributes with opposite sign, and double the magnitude, of the radiation term and $\ddot{a} > 0$.

Summary.

The standard cosmology based on a Big Bang augmented by an inflationary era is impressively consistent with the detailed data from WMAP3 when dark energy, most conservatively a cosmological constant, is included. Our objections to this standard model are more aesthetic than motivated directly by observations. The first objection is the nature of the initial singularity and the initial conditions. A second objection, not of concern to all colleagues, is that the predicted fate of the universe is an infinitely long expansion.

We have outlined here a cyclic cosmology resting on phantom dark energy where these objections are ameliorated: the classical density and temperature never become infinite and future expansion is truncated. Also, our proposal of deflation naturally leads to a multiverse picture, somewhat reminiscent of that predicted in eternal inflation, though here the proliferation of universes must be infinite and originates at the opposite end of a cyclic cosmology, at its maximum rather than at its minimum size.

We present this cyclic universe proposal mainly in the hope that it will stimulate an improved and more consistent formulation by others.

Thank you for your attention