Renormalization and Yang-Mills with Lorentz Violation

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Overview of Talk

- Introduction to Yang-Mills and Lorentz Violation
- Feynman Rules
- One-Loop CPT-Even Results
- One-Loop CPT-Odd Results
- Beta Functions
- Summary
Introduction to Yang-Mills

Constructed using Lie-Algebra valued vector potential $A^\mu$ to form covariant derivative $D^\mu = \partial^\mu + igA^\mu$ and gauge covariant curvature

$$F^{\mu\nu} = -\frac{i}{g} [D^{\mu}, D^{\nu}]$$

Pure Yang-Mills action given by

$$S_{YM} = - \frac{1}{2} \int d^4 x Tr F_{\mu\nu} F^{\mu\nu}$$

Impose gauge fixing using Faddeev-Popov procedure → introduce ghost fields $c$

$$S_{FP} = - \frac{1}{2} \int d^4 x \left[ Tr F_{\mu\nu} F^{\mu\nu} + \frac{1}{2\xi} (\partial_\mu A^\mu)^2 + \bar{c}(-\partial_\mu D^\mu)c \right]$$
Motivation and Implementation of Lorentz Violation

- Generic theories underlying standard model "naturally" allow for Lorentz-breaking effects: Original conception in string theory
  - Kostelecky and Samuel 1989

- Incorporate into standard model using effective field theory to generate Standard Model Extension (SME)
  - DC and Kostelecky 1997

⇒ advantage of model independent framework
One way of implementation: Spontaneous Symmetry Breaking

- Lehnert hep-ph/0611177
Including Lorentz Violation into Yang-Mills

If want to maintain gauge invariance and power-counting renormalizability, can include

\[ \mathcal{L}_{CPT-even} = -\frac{1}{2} (k_F)_{\mu\nu\alpha\beta} Tr F^{\mu\nu} F^{\alpha\beta} \]

\[ \mathcal{L}_{CPT-odd} = -\frac{1}{2} (k_{AF})^{\kappa}_{\epsilon} Tr (A^\lambda F^{\mu\nu} - \frac{2}{3} i g A^\lambda A^\mu A^\nu) \]

In this expression, \((k_F)_{\mu\nu\alpha\beta}\) and \((k_{AF})^{\kappa}\) are constant tensor background fields that break Lorentz invariance

(The \(k_F\) terms are required to be trace free: \(g_{\mu\alpha} (k_F)_{\mu\nu\alpha\beta} = 0\) for calculational simplicity)

Doing this for all fields \(\rightarrow\) Standard Model Extension (SME)
Some related work on renormalization in SME

- One Loop Renormalization of Lorentz-violating QED
  Kostelecky, Lane, and Pickering, PRD 2002

- Lorentz-violating QED renormalization in curved space
  Barredo-Peixoto and Shapiro, PLB 2006

- Lorentz violation and Faddeev-Popov Ghosts
  Altschul, PRD 2006

- New Bounds on Isotropic Lorentz Violation
  Carone, Sher, and Vanderhaeghen, PRD 2006
Feynman Rules for Conventional Terms

\[ \mu, a \to \nu, b = -i\delta^{ab}(g^{\mu\nu} - (1 - \xi)\frac{q^\mu q^\nu}{q^2})/q^2 \]

\[ = -gf^{abc}[g^{\mu\nu}(k - p)^\rho + g^{\nu\rho}(p - q)^\mu + g^{\rho\mu}(q - k)^\nu] \]

\[ = -ig^2[f^{abef}\epsilon^{cde}(g^{\mu\rho}g^{\nu\sigma} - g^{\mu\sigma}g^{\nu\rho}) + \text{perms}] . \]
Feynman Rules for CPT-even terms

\[ \mu, a \cdots \nu, b = -2i\delta^{ab}k_F^{\alpha\mu\beta\nu}q_\alpha q_\beta \]

\[ \triangleright = 2gf^{abc}[k_F^{\alpha\mu\nu\rho}k_\alpha + k_F^{\alpha\nu\rho\mu}p_\alpha + k_F^{\alpha\rho\mu\nu}q_\alpha] = 2g(V_k)^{abc}_{\mu\nu\rho} \]

\[ \times = -2ig^2[f^{abe}f^{cde}k_F^{\mu\nu\rho\sigma} + \text{perms}] = -2ig^2(V_k)^{abcd}_{\mu\nu\rho\sigma} , \]
Feynman Rules for CPT-odd terms

\[ \mu, a \otimes \otimes \nu, b = \delta^{ab}(k_{AF})^\kappa \epsilon_{\kappa \mu \beta \nu} p^\beta, \]

\[ = igf^{abc}(k_{AF})^\kappa \epsilon_{\kappa \mu \nu \rho}. \]
Example: conventional gluon two-point function evaluated using dimensional regularization

\[
\frac{1}{2} \underbrace{\text{tadpole}} + \underbrace{\text{tadpole}} = (\frac{5}{3} + \frac{1}{2}(1 - \xi))i(q^2 g^{\mu\nu} - q^\mu q^\nu)\delta^{ab}\tilde{g}^2
\]

where

\[
\tilde{g}^2 = \frac{g^2}{(4\pi)^2}C_2(G)\Gamma(2 - \frac{d}{2})
\]

with \(C_2(G)\) the quadratic Casimir element of adjoint rep
Divergence absorbed using counter term

\[ \mathcal{L}_{ct} = -\frac{1}{2} \delta_3 \left( \partial^\mu A^{a\nu} \partial_\mu A^a_{\nu} - \partial^\nu A^{a\mu} \partial_\mu A^a_{\nu} \right) \]

with

\[ \delta_3 = \left( \frac{5}{3} + \frac{1}{2} (1 - \xi) \right) \tilde{g}^2 = Z_3 - 1 \]

where \( Z_3 \) is multiplicative field renormalization factor \( A_B = Z_3^{1/2} A \)

Corresponding three-point vertex implies renormalization of bare coupling \( g_B = Z_g g \) with

\[ Z_g = 1 - \frac{11}{6} \tilde{g}^2 \]
1-Loop results for CPT-even terms

\[ \mathcal{L}_{\text{LVE}} = (k_F)_{\mu\nu\alpha\beta} \left[ -\partial^\mu A^{a\nu} \partial^\alpha A^{a\beta} + gf^{abc} (\partial^\mu A^{c\nu}) A^{a\alpha} A^{b\beta} ight. \\
- \left. \frac{1}{4} g^2 f^{abe} f^{cde} A^{a\mu} A^{b\nu} A^{c\alpha} A^{d\beta} \right] \]

Feynman rules for two-point function

\[ \mu, a \cdots \nu, b = -2i \delta^{ab} k_F^{\alpha\mu\beta\nu} q_\alpha q_\beta \]

and three-point function

\[ = 2gf^{abc} \left[ k_F^{\alpha\mu\nu\rho} k_\alpha + k_F^{\alpha\nu\rho\mu} p_\alpha + k_F^{\alpha\rho\mu\nu} q_\alpha \right] = 2g(V_k)^{abc}_{\mu\nu\rho} \]
Two-point function calculation

\[
\frac{1}{2} \quad \includegraphics[width=0.1\textwidth]{diagram1} + \frac{1}{2} \quad \includegraphics[width=0.1\textwidth]{diagram2} = (\frac{28}{3} - (1 - \xi)) i \delta^{ab} \tilde{g}^2 k_F^{\alpha\mu\beta\nu} q_\alpha q_\beta.
\]

\[
\quad \includegraphics[width=0.1\textwidth]{diagram3} = (-\frac{4}{3} + 2(1 - \xi)) i \delta^{ab} \tilde{g}^2 k_F^{\alpha\mu\beta\nu} q_\alpha q_\beta.
\]

Combining the above diagrams yields the two-point counter-term

\[
-\delta_{k_F}^{2g}(k_F)_{\mu\nu\alpha\beta} \partial^\mu A^{a\nu} \partial^\alpha A^{a\beta}
\]

with

\[
\delta_{k_F}^{2g} = (4 + \frac{1}{2}(1 - \xi)) \tilde{g}^2,
\]
Assuming multiplicative renormalization \((k_F)_B = Z_{k_F} k_F\) implies:

\[
Z_{k_F} = 1 + \frac{7}{3}\tilde{g}^2
\]

\(Z_{k_F}\) is only adjustable parameter, so three- and four-point radiative corrections are fixed (\(Z_3\) and \(Z_g\) fixed by conventional terms)
Three-point function calculation

\[
\begin{align*}
= & \left(3 - 3(1 - \xi)\right) \tilde{g}^2 (V_k)_{\mu\nu\rho}^{abc} , \\
= & \left(-\frac{3}{4} + \frac{3}{4}(1 - \xi)\right) \tilde{g}^2 (V_k)_{\mu\nu\rho}^{abc} ,
\end{align*}
\]

\[
\frac{1}{2} + \text{cross terms} = \left(-9 + \frac{9}{4}(1 - \xi)\right) \tilde{g}^2 (V_k)_{\mu\nu\rho}^{abc} ,
\]

\[
+ \text{cross terms} = \left(\frac{3}{4} - \frac{3}{2}(1 - \xi)\right) \tilde{g}^2 (V_k)_{\mu\nu\rho}^{abc} .
\]

Recall: \((V_k)_{\mu\nu\rho}^{abc}\) is lowest-order 3-point vertex and

\[
\tilde{g}^2 = \frac{g^2}{(4\pi)^2} C_2(G) \Gamma\left(2 - \frac{d}{2}\right)
\]
All are of the form of three-point counter-term

\[ \delta^3 g_{k_F(k_F)}^{\mu\nu\alpha\beta} g f^{abc} (\partial^\mu A^{cv}) A^{a\alpha} A^{b\beta} \]

Three-point calculation fixes

\[ \delta^3 g_{k_F} = (3 + \frac{3}{4}(1 - \xi)) \tilde{g}^2 \]

This is consistent with

\[ \delta^3 g_{k_F} = Z_g Z_3^{3/2} Z_{k_F} - 1 \]

as is required for multiplicative renormalization
An analogous calculation of the four-point function (performed in $\xi = 1$ gauge for calculational simplicity) yields the counter-term

$$-rac{1}{4}g^2 \delta_{k_F}^{4g}(k_F)_{\mu\nu\alpha\beta} f^{abe} f^{cde} A^a_{\mu} A^b_{\nu} A^c_{\alpha} A^d_{\beta}$$

with

$$\delta_{k_F}^{4g} = 2\tilde{g}^2,$$

which is again consistent with the multiplicative renormalization prediction

$$\delta_{k_F}^{4g} = (Z_g Z_3)^2 Z_{k_F} - 1$$
One-loop multiplicative renormalization of CPT-even terms implemented using

\[ A_B = Z_3^{1/2} A, \quad g_B = Z_g g \]

together with

\[ (k_F)_B = Z_{k_F} k_F \]

where \( Z \) values are

\[ Z_3 = 1 + (5/3 + 1/2(1 - \xi))\bar{g}^2 \]

\[ Z_g = 1 + (11/6)\bar{g}^2 \]

\[ Z_{k_F} = 1 + (7/3)\bar{g}^2 \]

Note: \( Z_3 \) carries all gauge dependence as in conventional case.
1-Loop results for CPT-odd terms

\[ \mathcal{L}_{\text{LVO}} = -\frac{1}{2}(k_{AF})^\kappa \epsilon_{\kappa\lambda\mu\nu} A^{a\lambda} \partial^\mu A^{a\nu} + \frac{1}{6} g(k_{AF})^\kappa \epsilon_{\kappa\lambda\mu\nu} f^{abc} A^{a\lambda} A^{b\mu} A^{c\nu} \]

Feynman rules for these terms

\[ \mu, a \ldots \nu, b = \delta^{ab}(k_{AF})^\kappa \epsilon_{\kappa\mu\beta\nu} q^\beta, \]

\[ = ig f^{abc}(k_{AF})^\kappa \epsilon_{\kappa\mu\nu\rho}. \]

(Note: No correction to four-point vertex)
Correction to two-point function

→ Same diagrams as CPT-even calc requires counter-term

\[-\frac{1}{2}(k_{AF})^\kappa \epsilon_{\kappa \lambda \mu \nu} \delta_{k_{AF}}^{2g} A^{a \lambda} \partial^\mu A^{a \nu}\]

\[\delta_{k_{AF}}^{2g} = (-2 + \frac{1}{2}(1 - \xi))\tilde{g}^2\]

Gives renormalization factor \((k_{AF})^\kappa_B = Z_{k_{AF}} (k_{AF})^\kappa\) with

\[Z_{k_{AF}} = 1 - \frac{11}{3} \tilde{g}^2\]
Correction to three-point function gives counter-term

\[ \frac{1}{2} (k_{AF})^\kappa \epsilon_{\kappa \lambda \mu \nu} \frac{1}{3} g \delta^{3g}_{k_{AF}} f^{abc} A^{a \lambda} A^{b \mu} A^{c \nu} \]

with

\[ \delta^{3g}_{k_{AF}} = (-3 + \frac{3}{4} (1 - \xi)) \tilde{g}^2 . \]

as required for successful multiplicative renormalization:

\[ \delta_{k_{AF}} = 1 + Z_{k_{AF}} Z g Z_{3}^{3/2} \]

Note: Four-point function unmodified, as expected
Beta Functions

One-Loop calculations yield the following beta functions

\[ \beta_g = -\frac{11g^3}{3(4\pi)^2}C_2(G) \]

and

\[ \beta_{k_F} = \frac{14g^2}{3(4\pi)^2}C_2(G)k_F, \quad \beta_{k_{AF}} = -\frac{22g^2}{3(4\pi)^2}C_2(G)k_{AF} \]

(Assumes validity of full renormalization program...)
Renormalization group equations solved using

\[ Q(\mu) = 1 + \frac{22g_0^2}{3(4\pi)^2}C_2(G)\ln\frac{\mu}{\mu_0} \]

to give usual running of coupling \( g^2(\mu) = Q^{-1}g_0^2 \)

Lorentz-Violating CPT-even coupling evolves as

\[ k_F = (k_F)_0 Q^{7/11} \]

while CPT-odd coupling behaves as

\[ k_{AF} = (k_{AF})_0 Q^{-1} \]

- CPT-odd terms are asymptotically free (run like \( g^2 \))

- CPT-even terms grow with energy scale
Summary

• Multiplicative renormalization is remarkably successful in full non-linear Yang-Mills theory with Lorentz violation at one loop

• Running indicates that CPT-even violations may be more significant than CPT-odd terms at higher energies in QCD

• More realistic model will incorporate fermions and other standard-model fields to deduce new bounds on physical parameters
Some upcoming Talks on Lorentz Violation at the Conference

Fri 11:00 - Matt Mewes
→ CMB Tests of Lorentz Invariance

Sat 3:00 - Brett Altschul
→ Limits on Lorentz Violation in High-Energy Astrophysics