Renormalization and Yang-Mills with Lorentz Violation

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Talk presented at Miami 2006

- DC and P. McDonald, hep-ph/0609084 to appear in PRD

Overview of Talk

- Introduction to Yang-Mills and Lorentz Violation
- Feynman Rules
- One-Loop CPT-Even Results
- One-Loop CPT-Odd Results
- Beta Functions
- Summary

Introduction to Yang-Mills

Constructed using Lie-Algebra valued vector potential A^μ to form covariant derivative $D^\mu = \partial^\mu + igA^\mu$ and gauge covariant curvature

$$F^{\mu\nu} = -\frac{i}{g}[D^\mu, D^\nu]$$

Pure Yang-Mills action given by

$$S_{YM} = -\frac{1}{2}\int d^4x Tr F_{\mu\nu}F^{\mu\nu}$$

Impose gauge fixing using Faddeev-Popov procedure
$\longrightarrow$ introduce ghost fields c

$$S_{FP} = -\frac{1}{2}\int d^4x \left[Tr F_{\mu\nu}F^{\mu\nu} + \frac{1}{2\xi}(\partial_\mu A^\mu)^2 + \bar{c}(-\partial_\mu D^\mu)c\right]$$

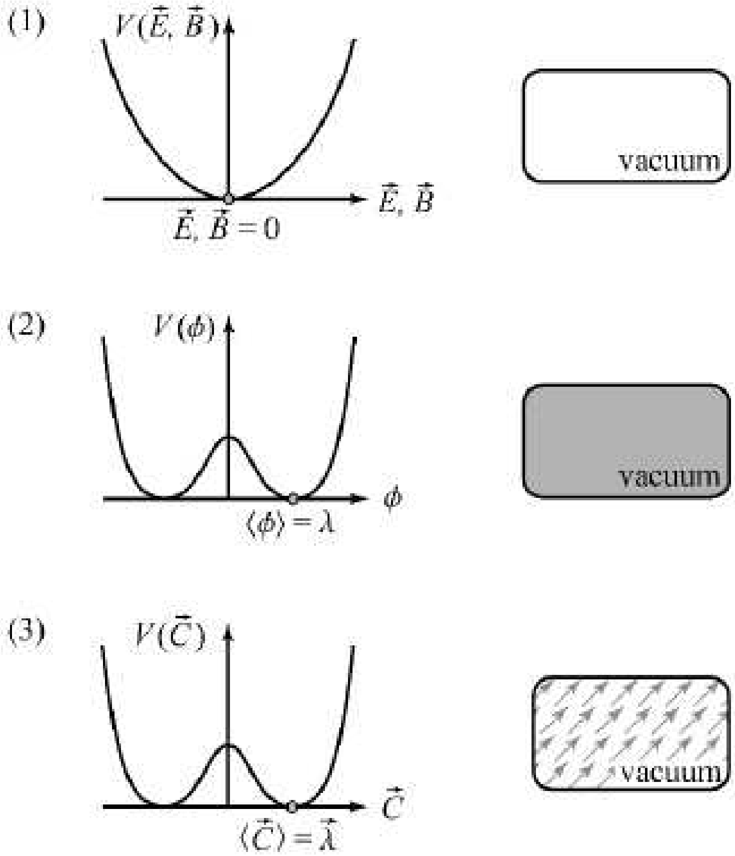
Motivation and Implementation of Lorentz Violation

- Generic theories underlying standard model ”naturally” allow for Lorentz-breaking effects: Original conception in string theory
  - Kostelecky and Samuel 1989

- Incorporate into standard model using effective field theory to generate Standard Model Extension (SME)
  - DC and Kostelecky 1997

⇒ advantage of model independent framework

One way of implementation: Spontaneous Symmetry Breaking



- Lehnert hep-ph/0611177

Including Lorentz Violation into Yang-Mills

If want to maintain gauge invariance and power-counting renormalizability, can include

$$\mathcal{L}_{CPT-even} = -\frac{1}{2}(k_F)_{\mu\nu\alpha\beta} Tr F^{\mu\nu} F^{\alpha\beta}$$

$$\mathcal{L}_{CPT-odd} = -\frac{1}{2}(k_{AF})^{\kappa}\epsilon_{\kappa\lambda\mu\nu} Tr(A^{\lambda}F^{\mu\nu} - \tfrac{2}{3}igA^{\lambda}A^{\mu}A^{\nu})$$

In this expression, $(k_F)^{\mu\nu\alpha\beta}$ and $(k_{AF})^{\kappa}$ are constant tensor background fields that break Lorentz invariance

(The k_F terms are required to be trace free: $g_{\mu\alpha}(k_F)^{\mu\nu\alpha\beta} = 0$ for calculational simplicity)

Doing this for all fields → Standard Model Extension (SME)

Some related work on renormalization in SME

- One Loop Renormalization of Lorentz-violating QED
  Kostelecky, Lane, and Pickering, PRD 2002

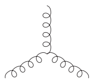
- Lorentz-violating QED renormalization in curved space
  Barredo-Peixoto and Shapiro, PLB 2006

- Lorentz violation and Faddeev-Popov Ghosts
  Altschul, PRD 2006

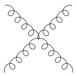
- New Bounds on Isotropic Lorentz Violation
  Carone, Sher, and Vanderhaeghen, PRD 2006

Feynman Rules for Conventional Terms

μ, a  $\nu, b = -i\delta^{ab}\left(g^{\mu\nu} - (1-\xi)\frac{q^\mu q^\nu}{q^2}\right)/q^2$



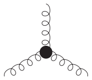
$$= -g f^{abc}[g^{\mu\nu}(k-p)^\rho + g^{\nu\rho}(p-q)^\mu + g^{\rho\mu}(q-k)^\nu]$$



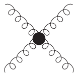
$$= -ig^2[f^{abe}f^{cde}(g^{\mu\rho}g^{\nu\sigma} - g^{\mu\sigma}g^{\nu\rho}) + \text{perms}]\ .$$

Feynman Rules for CPT-even terms

μ, a  $\nu, b = -2i\delta^{ab} k_F^{\alpha\mu\beta\nu} q_\alpha q_\beta$



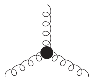
$$= 2gf^{abc}[k_F^{\alpha\mu\nu\rho} k_\alpha + k_F^{\alpha\nu\rho\mu} p_\alpha + k_F^{\alpha\rho\mu\nu} q_\alpha] = 2g(V_k)^{abc}_{\mu\nu\rho}$$



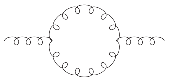
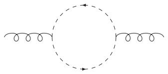
$$= -2ig^2[f^{abe}f^{cde}k_F^{\mu\nu\rho\sigma} + \text{perms}] = -2ig^2(V_k)^{abcd}_{\mu\nu\rho\sigma} \ ,$$

Feynman Rules for CPT-odd terms

μ, a  $\nu, b = \delta^{ab}(k_{AF})^{\kappa}\epsilon_{\kappa\mu\beta\nu}p^{\beta} \ ,$

 $= igf^{abc}(k_{AF})^{\kappa}\epsilon_{\kappa\mu\nu\rho} \ .$

Example: conventional gluon two-point function evaluated using dimensional regularization

$\frac{1}{2}$  $+$ tadpole $+$  $= (\frac{5}{3} + \frac{1}{2}(1-\xi))i(q^2 g^{\mu\nu} - q^\mu q^\nu)\delta^{ab}\tilde{g}^2$

where

$$\tilde{g}^2 = \frac{g^2}{(4\pi)^2} C_2(G)\Gamma(2 - \tfrac{d}{2})$$

with $C_2(G)$ the quadratic Casimir element of adjoint rep

Divergence absorbed using counter term

$$\mathcal{L}_{ct} = -\tfrac{1}{2}\delta_3 \left(\partial^\mu A^{a\nu}\partial_\mu A^a_\nu - \partial^\nu A^{a\mu}\partial_\mu A^a_\nu\right)$$

with

$$\delta_3 = (\tfrac{5}{3} + \tfrac{1}{2}(1-\xi))\tilde{g}^2 = Z_3 - 1$$

where Z_3 is multiplicative field renormalization factor $A_B = Z_3^{1/2} A$

Corresponding three-point vertex implies renormalization of bare coupling $g_B = Z_g g$ with

$$Z_g = 1 - \frac{11}{6}\tilde{g}^2$$

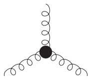
1-Loop results for CPT-even terms

$$\mathcal{L}_{\mathsf{LVE}} = (k_F)_{\mu\nu\alpha\beta}\left[-\partial^\mu A^{a\nu}\partial^\alpha A^{a\beta} + g f^{abc}(\partial^\mu A^{c\nu})A^{a\alpha}A^{b\beta} - \frac{1}{4}g^2 f^{abe}f^{cde}A^{a\mu}A^{b\nu}A^{c\alpha}A^{d\beta}\right]$$

Feynman rules for two-point function

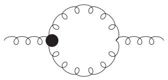
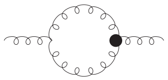
μ, a  $\nu, b = -2i\delta^{ab}k_F^{\alpha\mu\beta\nu}q_\alpha q_\beta$

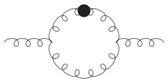
and three-point function



$$= 2gf^{abc}[k_F^{\alpha\mu\nu\rho}k_\alpha + k_F^{\alpha\nu\rho\mu}p_\alpha + k_F^{\alpha\rho\mu\nu}q_\alpha] = 2g(V_k)^{abc}_{\mu\nu\rho}$$

Two-point function calculation

$\frac{1}{2}$  $+ \frac{1}{2}$  $= \left(\frac{28}{3} - (1-\xi)\right) i\delta^{ab}\tilde{g}^{2} k_F^{\alpha\mu\beta\nu} q_\alpha q_\beta \ .$

 $= \left(-\frac{4}{3} + 2(1-\xi)\right) i\delta^{ab}\tilde{g}^{2} k_F^{\alpha\mu\beta\nu} q_\alpha q_\beta .$

Combining the above diagrams yields the two-point counter-term

$$-\delta^{2g}_{k_F} (k_F)_{\mu\nu\alpha\beta} \partial^\mu A^{a\nu} \partial^\alpha A^{a\beta}$$

with

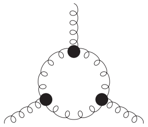
$$\delta^{2g}_{k_F} = \left(4 + \tfrac{1}{2}(1-\xi)\right)\tilde{g}^{2} \ ,$$

Assuming multiplicative renormalization $(k_F)_B = Z_{k_F} k_F$ implies:

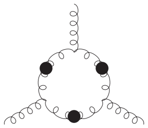
$$\boxed{Z_{k_F} = 1 + \frac{7}{3}\tilde{g}^2}$$

Z_{k_F} is only adjustable parameter, so three- and four-point radiative corrections are fixed (Z_3 and Z_g fixed by conventional terms)

Three-point function calculation

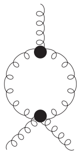


$$= (3 - 3(1-\xi))g\tilde{g}^2(V_k)^{abc}_{\mu\nu\rho}\ ,$$

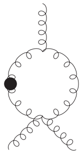


$$= (-\tfrac{3}{4} + \tfrac{3}{4}(1-\xi))g\tilde{g}^2(V_k)^{abc}_{\mu\nu\rho}\ ,$$

$\frac{1}{2}$



$$+ \text{cross terms} = (-9 + \tfrac{9}{4}(1-\xi))g\tilde{g}^2(V_k)^{abc}_{\mu\nu\rho}\ ,$$



$$+ \text{cross terms} = (\tfrac{3}{4} - \tfrac{3}{2}(1-\xi))g\tilde{g}^2(V_k)^{abc}_{\mu\nu\rho}\ .$$

Recall: $(V_k)^{abc}_{\mu\nu\rho}$ is lowest-order 3-point vertex and

$$\tilde{g}^2 = \frac{g^2}{(4\pi)^2}C_2(G)\Gamma(2-\tfrac{d}{2})$$

All are of the form of three-point counter-term

$$\delta^{3g}_{k_F}(k_F)_{\mu\nu\alpha\beta}\; g f^{abc}(\partial^{\mu}A^{c\nu})A^{a\alpha}A^{b\beta}$$

Three-point calculation fixes

$$\delta^{3g}_{k_F} = (3 + \tfrac{3}{4}(1-\xi))\tilde{g}^2$$

This is consistent with

$$\delta^{3g}_{k_F} = Z_g Z_3^{3/2} Z_{k_F} - 1$$

as is required for multiplicative renormalization

An analogous calculation of the four-point function (performed in $\xi = 1$ gauge for calculational simplicity) yields the counterterm

$$-\frac{1}{4}g^2\delta^{4g}_{k_F}(k_F)_{\mu\nu\alpha\beta}f^{abe}f^{cde}A^{a\mu}A^{b\nu}A^{c\alpha}A^{d\beta}$$

with

$$\delta^{4g}_{k_F} = 2\tilde{g}^2\ ,$$

which is again consistent with the multiplicative renormalization prediction

$$\delta^{4g}_{k_F} = (Z_g Z_3)^2 Z_{k_F} - 1$$

One-loop multiplicative renormalization of CPT-even terms implemented using

$$A_B = Z_3^{1/2} A \ , \quad g_B = Z_g g$$

together with

$$(k_F)_B = Z_{k_F} k_F$$

where Z values are

$$Z_3 = 1 + (5/3 + 1/2(1-\xi))\tilde{g}^2$$

$$Z_g = 1 + (11/6)\tilde{g}^2$$

$$Z_{k_F} = 1 + (7/3)\tilde{g}^2$$

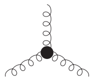
Note: Z_3 carries all gauge dependence as in conventional case

1-Loop results for CPT-odd terms

$$\mathcal{L}_{\mathrm{LVO}} = -\tfrac{1}{2}(k_{AF})^{\kappa}\epsilon_{\kappa\lambda\mu\nu}A^{a\lambda}\partial^{\mu}A^{a\nu} + \tfrac{1}{6}g(k_{AF})^{\kappa}\epsilon_{\kappa\lambda\mu\nu}f^{abc}A^{a\lambda}A^{b\mu}A^{c\nu}$$

Feynman rules for these terms

μ, a  $\nu, b = \delta^{ab}(k_{AF})^{\kappa}\epsilon_{\kappa\mu\beta\nu}q^{\beta}$,



$$= igf^{abc}(k_{AF})^{\kappa}\epsilon_{\kappa\mu\nu\rho} \ .$$

(Note: No correction to four-point vertex)

Correction to two-point function

→ Same diagrams as CPT-even calc requires counter-term

$$-\tfrac{1}{2}(k_{AF})^{\kappa}\epsilon_{\kappa\lambda\mu\nu}\delta^{2g}_{k_{AF}}A^{a\lambda}\partial^{\mu}A^{a\nu}$$

$$\delta^{2g}_{k_{AF}} = (-2 + \tfrac{1}{2}(1-\xi))\tilde{g}^2$$

Gives renormalization factor $(k_{AF})^{\kappa}_{B} = Z_{k_{AF}}(k_{AF})^{\kappa}$ with

$$\boxed{Z_{k_{AF}} = 1 - \tfrac{11}{3}\tilde{g}^2}$$

Correction to three-point function gives counter-term

$$\frac{1}{2}(k_{AF})^{\kappa}\epsilon_{\kappa\lambda\mu\nu}\frac{1}{3}g\delta^{3g}_{k_{AF}}f^{abc}A^{a\lambda}A^{b\mu}A^{c\nu}$$

with

$$\delta^{3g}_{k_{AF}} = (-3+\frac{3}{4}(1-\xi))\tilde{g}^2 \ .$$

as required for successful multiplicative renormalization:

$$\delta_{k_{AF}} = 1 + Z_{k_{AF}}Z_gZ_3^{3/2}$$

Note: Four-point function unmodified, as expected

Beta Functions

One-Loop calculations yield the following beta functions

$$\beta_g = -\frac{11g^3}{3(4\pi)^2}C_2(G)$$

and

$$\beta_{k_F} = \frac{14g^2}{3(4\pi)^2}C_2(G)k_F \ , \qquad \beta_{k_{AF}} = -\frac{22}{3}\frac{g^2}{(4\pi)^2}C_2(G)k_{AF}$$

(Assumes validity of full renormalization program...)

Renormalization group equations solved using

$$Q(\mu) = 1 + \frac{22 g_0^2}{3(4\pi)^2} C_2(G) \ln \frac{\mu}{\mu_0}$$

to give usual running of coupling $g^2(\mu) = Q^{-1} g_0^2$
Lorentz-Violating CPT-even coupling evolves as

$$k_F = (k_F)_0 Q^{7/11}$$

while CPT-odd coupling behaves as

$$k_{AF} = (k_{AF})_0 Q^{-1}$$

- CPT-odd terms are asymptotically free (run like g^2)
- CPT-even terms grow with energy scale

Summary

- Multiplicative renormalization is remarkably successful in full non-linear Yang-Mills theory with Lorentz violation at one loop

- Running indicates that CPT-even violations may be more significant than CPT-odd terms at higher energies in QCD

- More realistic model will incorporate fermions and other standard-model fields to deduce new bounds on physical parameters

Some upcoming Talks on Lorentz Violation at the Conference

Fri 11:00 - Matt Mewes
→ CMB Tests of Lorentz Invariance

Sat 3:00 - Brett Altschul
→ Limits on Lorentz Violation in High-Energy Astrophysics