Baryons and Skyrmions in Orientifold Field Theories

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In order to have a large $N$ limit with asymptotic freedom we can choose between $N_f$ Dirac quarks in the two-index symmetric $(S)\ Q_{\alpha\beta}^f$ or anti-symmetric $(A)\ Q_{[\alpha\beta]}^f$.

For 3 colors the two-index antisymmetric is equal to the anti-fundamental $e^{\gamma}_{Q_{\alpha\beta}} = \frac{1}{2} \epsilon^{\gamma\alpha\beta} Q_{[\alpha\beta]}$ (Corrigan-Ramond 79).

Planar Equivalence relates them in the large $N$ limit (Armoni-Shifman-Veneziano 03).
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QCD with Quarks in Higher Representations

- Ordinary QCD is $SU(N)$ Yang-Mills with $N_f$ Dirac quarks in the fundamental representation.
- In order to have a *large N limit* with *asymptotic freedom* we can choose between:
  - $N_f$ Dirac quarks in the **two-index symmetric** ($S$) $Q^\{\alpha \beta\}$ or **anti-symmetric** ($A$) $Q^{[\alpha \beta]}$
  - $N_f$ Majorana quarks in the **adjoint** representation $\lambda^\alpha_\beta$

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1. For 3 colors the two-index antisymmetric is equal to the anti-fundamental $\tilde{Q}_\gamma = \frac{1}{2} \epsilon_{\gamma\alpha\beta} Q^{[\alpha\beta]}$ (Corrigan-Ramond 79).
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The purpose of the talk is to explain and solve this apparent puzzle.
The spectrum is composed by **MESONS** $\bar{Q}_\alpha Q^\alpha$ and **BARYONS** $\epsilon_{\alpha_1...\alpha_N} Q^{\alpha_1} \ldots Q^{\alpha_N}$. 
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The low energy effective Lagrangian is

$$S_{\text{eff}} = \frac{1}{16} F^2_\pi \int d^4x \left\{ \text{Tr} \partial_\mu U \partial_\mu U^{-1} + \text{higher derivatives} \right\}$$

where $U(x) = \exp \left( \frac{i \pi(x)}{F_\pi} \right)$, where $\pi(x)$ is the Nambu-Goldstone boson matrix
The spectrum is composed by mesons $\bar{Q}_\alpha Q^\alpha$ and baryons $\epsilon_{\alpha_1...\alpha_N} Q^{\alpha_1} ... Q^{\alpha_N}$.

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The pions coupling constant scales like $F_\pi \sim \frac{1}{\sqrt{N}}$. 
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Orientifold QCD

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\[
\epsilon_{\alpha_1\alpha_2...\alpha_N} \epsilon_{\beta_1\beta_2...\beta_N} Q^{\alpha_1\beta_1} Q^{\alpha_2\beta_2} \ldots Q^{\alpha_N\beta_N}
\]

- The baryonic sector is instead very different. For example the simplest baryon is
The Skyrmion

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- It is stabilized by the homotopy group

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\[ \pi_3 \left( SU \left( N_f \right) \right) = \mathbb{Z} \]

- Skyrmion can acquire spin and baryon number from the WZW term

\[ \Gamma_{WZW} = -i \frac{n}{240 \pi^2} \int_{\mathcal{M}^5} \epsilon^{\mu \nu \rho \sigma \tau} \text{Tr} \left( \partial_{\mu} U U^{-1} \ldots \partial_{\tau} U U^{-1} \right) \]

where \( n \) is an integer
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where $n$ is an integer

- The baryon number of the Skyrmion is equal to $n$; its statistic is equal to $(-1)^n$
The Skyrmion is the Baryon on ordinary QCD

- In ordinary QCD $n = N$ and so the Skyrmion has the same quantum numbers of $\epsilon_{\alpha_1...\alpha_N} Q^{\alpha_1} \ldots Q^{\alpha_N}$
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- The baryon is **ANTI-SYMMETRIC** in the gauge wave function

$$\psi_{\text{gauge}} \quad \psi_{\text{spin/flavor}} \quad \psi_{\text{space}}$$

- $-$  
- $+$  
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- In the large $N$ limit it can be approximated by a system of free bosons in a mean field potential $V_{\text{mean}}(r)$ created by the quarks themselves; the ground state is a Bose-Einstein condensate
The 1-body contribution is simply $N$ times the mass of the single quark.
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The 2-body interaction is of order $\frac{1}{N}$ but an additional combinatorial factor $\binom{N}{2}$ is needed and we obtain a contribution to the energy of order $N$.
The Skyrmion is not the simplest baryon on orientifold QCD

- In orientifold QCD $n = \frac{N(N+1)}{2}$ and so the Skyrmion does not have the same quantum numbers of $\epsilon_{\alpha_1\alpha_2...\alpha_N} \epsilon_{\beta_1\beta_2...\beta_N} Q^{\alpha_1\beta_1} Q^{\alpha_2\beta_2} ... Q^{\alpha_N\beta_N}$.
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- The mass of the Skyrmion is \( \propto N^2 \).
- The baryon is symmetric in the gauge wave function.

\[ \psi_{\text{gauge}} \psi_{\text{spin/flavor}} \psi_{\text{space}} \]

\[ + \quad + \quad - \]
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- In the large \( N \) limit it can be approximated by a system of free fermion in a mean field potential \( V_{\text{mean}}(r) \) created by the quarks themseves; the ground state is a Fermi degenerate gas
The Skyrmion is not the simplest baryon on orientifold QCD

- In orientifold QCD $n = \frac{N(N\pm 1)}{2}$ and so the Skyrmion does not have the same quantum numbers of $\epsilon_{\alpha_1 \alpha_2 \ldots \alpha_N}\epsilon_{\beta_1 \beta_2 \ldots \beta_N} Q^{\alpha_1 \beta_1} Q^{\alpha_2 \beta_2} \ldots Q^{\alpha_N \beta_N}$
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- In the large $N$ limit it can be approximated by a system of free fermion in a mean field potential $V_{\text{mean}}(r)$ created by the quarks themseves; the ground state is a Fermi degenerate gas
- The mass of the baryon is for sure not greater than the Fermi zero temperature pressure $\propto N^{4/3}$
Solution of the puzzle

- **The Skymion must be identified with a baryon that contains** \( \frac{N(N\pm1)}{2} \) **quarks**
Solution of the puzzle

- The Skymion must be identified with a baryon that contains $\frac{N(N\pm1)}{2}$ quarks.
- The Skymion is the "preferred" baryonic state since it minimizes the mass per unit of baryon number.
We are looking for a baryon with $\frac{N(N+1)}{2} = 3$ quarks
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The baryon is

\[ \epsilon_{\alpha_2 \alpha_1} \epsilon_{\beta_2 \alpha_3} \epsilon_{\beta_1 \beta_3} \ Q^{\{\alpha_1 \beta_1\}} Q^{\{\alpha_2 \beta_2\}} Q^{\{\alpha_3 \beta_3\}} \]
Proposition

There is one and only one gauge wave function that is a gauge singlet and completely antisymmetric under exchange of two quarks. This wave function is composed by \( \frac{N(N+1)}{2} \) quarks \( Q^{\{\alpha\beta\}} \) and is the completely antisymmetric subspace of the tensor product of \( \frac{N(N+1)}{2} \) quarks \( Q^{\{\alpha\}} \).

1. \( q^{\alpha_i} \) and \( q^{\beta_i} \) cannot belong to the same saturation line

2. If \( q^{\alpha_i} \) and \( q^{\alpha_j} \) belong to the same saturation line, the two partners \( q^{\beta_i} \) and \( q^{\beta_j} \) cannot belong to the same saturation line.
Another Example
Anti-symmetric representation 3 colors

- We are looking for a baryon with \( \frac{N(N-1)}{2} = 3 \) quarks
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The antisymmetric representation for \( N = 3 \) is equivalent to the anti-fundamental \( \bar{Q}_\gamma = \frac{1}{2}\epsilon_{\gamma\alpha\beta} Q^{[\alpha\beta]} \) and we know how to write a baryon for the anti-fundamental representation \( \epsilon^{\gamma\rho\tau} \bar{Q}_\gamma \bar{Q}_\rho \bar{Q}_\tau \)
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- The antisymmetric representation for \( N = 3 \) is equivalent to the anti-fundamental \( \tilde{Q}_\gamma = \frac{1}{2} \epsilon_{\gamma\alpha\beta} Q^{[\alpha\beta]} \) and we know how to write a baryon for the anti-fundamental representation \( \epsilon^{\gamma\rho\tau} \tilde{Q}_\gamma \tilde{Q}_\rho \tilde{Q}_\tau \)
- Substituting the relation between \( \tilde{Q}_\gamma \) and \( Q^{[\alpha\beta]} \) we obtain

\[
\frac{1}{2} ( \epsilon_{\gamma_1 \delta_1 \alpha} \epsilon_{\gamma_2 \delta_2 \beta} - \epsilon_{\gamma_2 \delta_2 \alpha} \epsilon_{\gamma_1 \delta_1 \beta} ) Q^{[\alpha\beta]} Q^{[\gamma_1 \delta_1]} Q^{[\gamma_2 \delta_2]} 
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1. One saturation line can contain at most one quark of the type \( Q^{[\gamma_i\delta_i]} \).

2. If \( q^{\alpha_i} \) and \( q^{\alpha_j} \) belong to the same saturation line, the two partners \( q^{\beta_i} \) and \( q^{\beta_j} \) cannot belong to the same saturation line.
We propose now a toy model to schematize the fundamental baryon

\[ \epsilon_{\alpha_1 \alpha_2 \ldots \alpha_N} \epsilon_{\beta_1 \beta_2 \ldots \beta_N} Q^{\alpha_1 \beta_1} Q^{\alpha_2 \beta_2} \ldots Q^{\alpha_N \beta_N} \]
A Toy model for the baryon

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- We have \( N \) quarks and 2 baryon vertices. Every quark is attached to two fundamental strings and every baryon vertex to \( N \) fundamental strings.
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- So the mean potential is
  \[ V_{\text{mean}}(R) = 2T_{\text{string}} |R| \]
Mass versus N dependence
Simplest baryon

- We have to fill the energy levels up to the Fermi surface

\[ Z R_F Z P_F d^3 R d^3 P (2\pi)^3 (P + V_{\text{mean}}(R)) = E \]

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We have to fill the energy levels up to the Fermi surface.

We indicate as $R_F$ and $P_F$ respectively the Fermi radius and momentum. The total energy and the number of quarks $N$ are given by

$$\int_{R_F}^{P_F} \int_{R}^{P} \frac{d^3 R d^3 P}{(2\pi)^3} (P + V_{\text{mean}}(R)) = E$$

$$\int_{R_F}^{P_F} \int_{R}^{P} \frac{d^3 R d^3 P}{(2\pi)^3} = N$$
The second equation gives a relation between the Fermi momentum and the Fermi radius, namely $P_F \sim N^{1/3}/R_F$. 
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Minimizing we obtain

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M_{N-Baryon} \sim N^{7/6} \sqrt{T_{\text{string}}}
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**The mass per unit of baryon number grows as $N^{1/6}$!**
At zero order the mass is $\propto N^2$

At first order we are tempted to say $g^2 \left( \frac{N^2}{2} \right) \propto N^3$

The correct answer is $g^2 \left( \frac{N^2}{2} \right) \propto N^2$ (Cherman-Cohen 06)

The mass per unit of baryon number is constant!
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identification still works, even if in a more subtle way.

- The orientifold large $N$ limit is a promising tool to study real 3-colors QCD.
Conclusion

- We have studied the baryon sector of orientifold QCD and showed that the
  \[ \text{Baryon} = \text{Skyrmion} \]
  identification still works, even if in a more subtle way
- The orientifold large $N$ limit is a promising tool to study real 3-colors QCD
- Beyond SM physics
Future works...

- Type 0A/B String theory realization
Future works...

1. Type 0A/B String theory realization
2. Skyrmion in adjoint QCD