Liénard's power formula The rate at which energy leaves the charge is

$$\frac{dW}{dt_{ret}} = \frac{1}{(\partial t_{ret}/\partial t)}\left(\frac{dW}{dt}\right) = \kappa\,\frac{dW}{dt}$$

with

$$\frac{\partial t_{ret}}{\partial t} = \frac{1}{\kappa} \equiv \left.\frac{1}{1-\hat{R}\cdot\overrightarrow{v}/c}\right|_{ret}$$

The latter follows from

$$t - t_{ret} = R/c \equiv \left|\overrightarrow{r} - \overrightarrow{s}\,(t_{ret})\right|/c$$

where $\overrightarrow{s}\,(t)$ is the particle's trajectory, and also $\overrightarrow{v}\,(t) = d\overrightarrow{s}/dt$. Explicitly

$$\begin{aligned}
\frac{\partial}{\partial t}(t-t_{ret}) &= 1-\left(\frac{\partial t_{ret}}{\partial t}\right) \\
\frac{\partial}{\partial t}\left|\overrightarrow{r} - \overrightarrow{s}\,(t_{ret})\right|/c &= \left(\frac{\partial t_{ret}}{\partial t}\right)\frac{\partial}{\partial t_{ret}}\left|\overrightarrow{r} - \overrightarrow{s}\,(t_{ret})\right|/c \\
&= \left(\frac{\partial t_{ret}}{\partial t}\right)\frac{\left(-\overrightarrow{r} + \overrightarrow{s}\,(t_{ret})\right)\cdot\overrightarrow{v}\,(t_{ret})/c}{\left|\overrightarrow{r} - \overrightarrow{s}\,(t_{ret})\right|} \\
&= -\left(\frac{\partial t_{ret}}{\partial t}\right)\left.\hat{R}\cdot\overrightarrow{v}/c\right|_{ret}
\end{aligned}$$

The angular distribution of radiated power is

$$\begin{aligned}
\frac{d\mathcal{P}}{d\Omega} &= \kappa R^2\left(\hat{R}\cdot\overrightarrow{S}_{rad}\right) = \frac{1}{\mu_0 c}\,\kappa R^2\left|\overrightarrow{E}_{rad}\right|^2 \\
&= \frac{1}{4\pi}\left(\frac{q^2}{4\pi\varepsilon_0 c^3}\right)\frac{1}{\kappa^5}\left|\hat{R}\times\left(\left(\hat{R}-\overrightarrow{v}/c\right)\times\overrightarrow{a}\right)\right|^2
\end{aligned}$$

Integrating over all angles leads to the total power radiated (Liénard's formula).

$$\begin{aligned}
\mathcal{P} &= \frac{2}{3}\left(\frac{q^2}{4\pi\varepsilon_0 c^3}\right)\gamma^6\left\{a^2 - \frac{1}{c^2}\left|\overrightarrow{v}\times\overrightarrow{a}\right|^2\right\} \\
&= \frac{2}{3}\left(\frac{q^2}{4\pi\varepsilon_0 c^3}\right)\left\{\gamma^4 a^2 + \gamma^6\left(\overrightarrow{a}\cdot\overrightarrow{v}/c\right)^2\right\}
\end{aligned}$$

where $\gamma = 1/\sqrt{1-v^2/c^2}$ is the ubiquitous Lorentz factor.

Some details: To carry out the angular integrations, we first resolve the square of the double cross-product as

$$\left|\hat{R}\times\left(\left(\hat{R}-\overrightarrow{v}/c\right)\times\overrightarrow{a}\right)\right|^2=\kappa^2a^2+2\kappa\left(\hat{R}\cdot\overrightarrow{a}\right)\left(\overrightarrow{a}\cdot\overrightarrow{v}/c\right)-\frac{1}{\gamma^2}\left(\hat{R}\cdot\overrightarrow{a}\right)^2$$

Then evaluate

$$\begin{aligned}
\mathcal{I} &= \int d\Omega\;\frac{1}{\kappa^3}=2\pi\int_{-1}^{+1}d\left(\cos\theta\right)\frac{1}{\left(1-\frac{v}{c}\cos\theta\right)^3}\\
&= \frac{\pi c}{v}\left(\frac{1}{\left(1-\frac{v}{c}\right)^2}-\frac{1}{\left(1+\frac{v}{c}\right)^2}\right)\\
&= 4\pi\frac{1}{\left(1-v^2/c^2\right)^2}\\
&= 4\pi\gamma^4
\end{aligned}$$

as well as

$$\begin{aligned}
\overrightarrow{\mathcal{I}} &= \int d\Omega\;\frac{1}{\kappa^4}\;\hat{R}=\frac{\overrightarrow{v}}{v^2}\;2\pi\int_{-1}^{+1}d\left(\cos\theta\right)\frac{v\cos\theta}{\left(1-\frac{v}{c}\cos\theta\right)^4}\\
&= \frac{\overrightarrow{v}}{v}\frac{1}{3}\frac{d\mathcal{I}}{d\left(\frac{v}{c}\right)}=\frac{4\pi\overrightarrow{v}}{3v}\left(4\;\frac{v}{c}\gamma^6\right)\\
&= \frac{16\pi}{3c}\gamma^6\overrightarrow{v}
\end{aligned}$$

using $d\gamma/d\left(\frac{v}{c}\right)=\frac{v}{c}\gamma^3$. Finally, there is

$$\int d\Omega\;\frac{1}{\kappa^5}\;\hat{R}_i\hat{R}_j\equiv a\;\delta_{ij}+b\;v_iv_j/v^2$$

with

$$\begin{aligned}
\left(a+b\right)v^2 &= v_iv_j\int d\Omega\;\frac{1}{\kappa^5}\;\hat{R}_i\hat{R}_j\\
&= \int d\Omega\;\frac{v^2\cos^2\theta}{\kappa^5}\\
&= \frac{4\pi v^2}{3}\frac{1+5v^2/c^2}{\left(1-v^2/c^2\right)^4}\\
&= 4\pi v^2\left(\frac{1}{3}+\frac{5}{3}\;v^2/c^2\right)\gamma^8
\end{aligned}$$

and

$$\begin{aligned} 3a+b &= \delta_{ij} \int d\Omega \; \frac{1}{\kappa^5} \; \hat{R}_i \hat{R}_j \\ &= \int d\Omega \; \frac{1}{\kappa^5} \\ &= \frac{2\pi c}{4v} \left(\frac{1}{\left(1-\frac{v}{c}\right)^4} - \frac{1}{\left(1+\frac{v}{c}\right)^4} \right) \\ &= 4\pi \left(1 + v^2/c^2\right) \gamma^8 \end{aligned}$$

That is to say,

$$a = \frac{1}{3} \left(1 - v^2/c^2\right) \; 4\pi\gamma^8 \; , \quad b = \frac{2v^2}{c^2} \; 4\pi\gamma^8$$

so

$$\int d\Omega \; \frac{1}{\kappa^5} \; \hat{R}_i \hat{R}_j = 4\pi \left(\frac{1}{3} \; \delta_{ij} \gamma^6 + \frac{2}{c^2} \; v_i v_j \gamma^8 \right)$$

Thus we have

$$\begin{aligned} \int d\Omega \; \frac{1}{\kappa^5} \left| \hat{R} \times \left(\left(\hat{R} - \overrightarrow{v}/c \right) \times \overrightarrow{a} \right) \right|^2 &= \int d\Omega \; \left\{ \frac{1}{\kappa^3} a^2 + \frac{2}{\kappa^4} \left(\hat{R} \cdot \overrightarrow{a} \right) \left(\overrightarrow{a} \cdot \overrightarrow{v}/c \right) - \frac{1}{\gamma^2 \kappa^5} \left(\hat{R} \cdot \overrightarrow{a} \right)^2 \right\} \\ &= 4\pi\gamma^4 a^2 + 2 \left(\overrightarrow{a} \cdot \overrightarrow{v}/c \right)^2 \frac{16\pi}{3} \; \gamma^6 - 4\pi \left(\frac{1}{3} \; a^2 \gamma^4 + 2 \left(\overrightarrow{a} \cdot \overrightarrow{v}/c \right)^2 \gamma^6 \right) \\ &= \frac{8\pi}{3} \left\{ \gamma^4 a^2 + \gamma^6 \left(\overrightarrow{a} \cdot \overrightarrow{v}/c \right)^2 \right\} \\ &= \frac{8\pi}{3} \; \gamma^6 \left\{ a^2 - \frac{1}{c^2} \left| \overrightarrow{v} \times \overrightarrow{a} \right|^2 \right\} \end{aligned}$$

This gives Liénard's result.