HW Problem 9: Assume Dirichlet boundary conditions on the L-shaped boundary \( \{ x \geq 0, t = 0 \} \cup \{ x = 0, t > 0 \} \), and solve \( \frac{\partial}{\partial t} T(x,t) = \lambda \frac{\partial^2}{\partial x^2} T(x,t) \) with the following boundary values. Keep the vertical part of the boundary at a fixed constant temperature, say \( T = 0 \), and consider the various cases:

\[
\begin{align*}
T(x > 0, t = 0) &= T_0 \quad \text{a constant} \\
T(x > 0, t = 0) &= T_0 e^{-x/L} \\
T(x > 0, t = 0) &= T_0 e^{-(x-a)^2/L^2}
\end{align*}
\]

HW Problem 10: Assume Neumann boundary conditions on the vertical part of the L-shaped boundary \( \{ x \geq 0, t = 0 \} \cup \{ x = 0, t > 0 \} \), specifically \( \frac{\partial}{\partial x} T(x, t > 0) |_{x=0} = 0 \), and solve \( \frac{\partial}{\partial t} T(x, t) = \lambda \frac{\partial^2}{\partial x^2} T(x, t) \) with the following Dirichlet conditions on the horizontal part.

\[
\begin{align*}
T(x > 0, t = 0) &= T_0 \quad \text{a constant} \\
T(x > 0, t = 0) &= T_0 e^{-x/L} \\
T(x > 0, t = 0) &= T_0 e^{-(x-a)^2/L^2}
\end{align*}
\]

HW Problem 11: For the “slab problem,” with \( T(x, t > 0) |_{x=0,L} = 0 \), \( T(0 < x < L, t = 0) = T_0 \), show that

\[
T(x, t) = \int_0^L g_D(x, X; t) T(X, 0) \, dX
\]

where

\[
g_D(x, X; t) = \frac{1}{2L} \left\{ \vartheta \left( \frac{\sqrt{\pi x} - X}{L}, \frac{\pi \lambda t}{L^2} \right) - \vartheta \left( \frac{\sqrt{\pi x} + X}{L}, \frac{\pi \lambda t}{L^2} \right) \right\}
\]

gives the result in Nearing, §10.2, Eqn(14). (Our definition of the two-variable \( \vartheta \) was given in HW Problem 5.)

HW Problem 12: Under what conditions is it true that the total “heat energy” within an interval \( a < x < b \) is conserved, i.e. that

\[
\frac{d}{dt} \int_a^b T(x, t) \, dx = 0 \quad ?
\]