HW Problem 22:
Consider two Gaussians on phase-space. Use the integral form of the star product
\[
f(x,p) \ast g(x,p) = \int \frac{dx_1 dp_1}{\pi \hbar} \int \frac{dx_2 dp_2}{\pi \hbar} f(x+x_1, p+p_1) g(x+x_2, p+p_2) \exp \left( \frac{2i}{\hbar} (x_1 p_2 - x_2 p_1) \right)
\]
to show
\[
\exp \left( -\frac{a}{\hbar} (x^2 + p^2) \right) \ast \exp \left( -\frac{b}{\hbar} (x^2 + p^2) \right) = \frac{1}{1 + ab} \exp \left( -\frac{a + b}{(1 + ab) \hbar} (x^2 + p^2) \right)
\]
Thus, Gaussians are exceptional and star commute with one another.

HW Problem 23:
Determine the time evolution of phase-space distributions for free particles, \( H = p^2/2 \), and for particles in a linear potential, \( H = p^2/2 + \lambda x \). That is to say, for these two cases solve the Wigner-Moyal equations.
\[
\frac{i\hbar}{\partial t} f(x,p;t) = H \ast f(x,p;t) - f(x,p;t) \ast H
\]
The star products here are most easily dealt with using the differential form:
\[\ast \equiv e^{\frac{i\hbar}{\pi} (\Delta_x \Delta_p - \Delta_p \Delta_x)}\]. The resulting PDEs may be solved through the use of the method of characteristics. Show that the \( \lambda \to 0 \) limit of your linear potential result coincides with your result for the free particle.

HW Problem 24:
Determine the phase-space energy-eigenfunctions for a particle in a linear potential \( H = p^2/2 + \lambda x \). That is, for real \( f_E(x,p) \), solve the pair of equations
\[
H \ast f_E(x,p) = E f_E(x,p) = f_E(x,p) \ast H
\]
What are the allowed values of \( E \)?

HW Problem 25:
Determine the evolution of \( u(x,t=0) = A e^{x/L} \) by formally summing the time power series solution for the Euler-Monge partial differential equation, \( \frac{\partial}{\partial t} u(x,t) = u(x,t) \frac{\partial}{\partial x} u(x,t) \). Show that
\[
u(x,t) = -\frac{L}{t} W \left( -\frac{At}{L} e^{x/L} \right)
\]
where \( W \) is Lambert’s function.