**HW Problem 20:**
Compute the generating function of the off-diagonal Wigner functions for the harmonic oscillator with Hamiltonian $H = \frac{1}{2} (p^2 + x^2)$. (Note that we have chosen units so that $M = \omega = \hbar = 1$.)
That is, compute the double sum
\[
\sum_{k=0}^{\infty} \sum_{n=0}^{\infty} \frac{(\sqrt{2}w)^k}{\sqrt{k!}} \frac{(\sqrt{2}t)^n}{\sqrt{n!}} f_{kn}(x, p)
\]
where
\[
f_{kn}(x, p) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{-ipy} \psi_k^*(x - \frac{1}{2}y) \psi_n(x + \frac{1}{2}y) \, dy
\]
and where the ortho-normalized wave functions
\[
\psi_n(x) = \frac{H_n(x) e^{-\frac{1}{2}x^2}}{\sqrt{2^n n! \sqrt{\pi}}}
\]
are generated by
\[
\sum_{n=0}^{\infty} \frac{(\sqrt{2}t)^n}{\sqrt{n!}} \psi_n(x) = \frac{1}{\pi^{1/4}} \exp \left( -\frac{1}{2}x^2 + 2tx - t^2 \right)
\]

**HW Problem 21:**
Use your answer to HW#20 to show that
\[
f_{kn}(x, p) = \frac{(-1)^k}{\pi} \sqrt{\frac{k!}{n!}} (4H)^{(n-k)/2} e^{(n-k) \arctan(p/x)} e^{-2H} L_k^{(n-k)}(4H)
\]
You may also find the following generating functions for the Laguerre polynomials to be useful.
\[
\sum_{n=0}^{\infty} t^n L_n^{(m)}(z) = \frac{1}{(1-t)^{m+1}} \exp \left( \frac{tz}{t-1} \right), \quad L_n(z) \equiv L_n^{(0)}(z)
\]
\[
\sum_{k=0}^{\infty} v^k L_k^{(n-k)}(z) = e^{-vz} (1+v)^n
\]
You do not have to derive these Laguerre identities.